4. Triangles

Exercise 4.1

1. Question

Fill in the blanks using the correct word given in brackets :

(i) All circles are……..(congruent, similar).

(ii) All squares are………(similar, congruent).

(iii) All……triangles are similar (isosceles, equilaterals).

(iv) Two triangles are similar, if heir corresponding angles are………(proportional, equal)

(v) Two triangles are similar, if their corresponding sides are………(proportional, equal)

(vi) Two polygons of the same number of sides are similar, if (a) their corresponding angles ae and (b) heir corresponding sides are………(equal, proportional)

Answer

(i) similar (ii) similar

(iii) equilateral (iv) equal

(v) proportional (vi) equal, proportional

2. Question

Write the truth value (T/F) of each of the following statements:

(i) Any two similar figures are congruent.

(ii) Any two congruent figures are similar.

(iii) Two polygons are similar, if their corresponding sides are proportional.

(iv) Two polygons are similar if their corresponding angles are proportional.

(v) Two triangles are similar if their corresponding sides are proportional.

(vi) Two triangles are similar if their corresponding angles are proportional.

Answer

(i) False (ii) True

(iii) False (iv) False

(v) True (vi) True

Exercise 4.2

1. Question

In a ΔABC, D and E are points on the sides AB and AC respectively such that $DE \parallel BC$

(i) If AD = 6 cm, DB = 9 cm and AE = 8 cm, find AC.

(ii) If $\frac{AD}{DB} = \frac{3}{4}$ and AC = 15 cm, find AE.

(iii) If $\frac{AD}{DB} = \frac{2}{3}$ and AC = 18 cm, find AE.

(iv) If AD = 4, AE = 8, DB = x − 4, and EC = 3x − 19, find x.

(v) If AD = 8 cm, AB = 12 cm and AE = 12 cm, find CE.

(vi) If AD = 4 cm, DB = 4.5 cm and AE = 8 cm, find AC.

(vii) If AD = 2 cm, AB = 6 cm and AC = 9 cm, find AE.

(viii) If $\frac{AD}{BD} = \frac{4}{5}$ and EC = 2.5 cm, find AE.

(ix) If AD = x, DB = x − 2, AE = x + 2 and EC = x − 1, find the value of x.

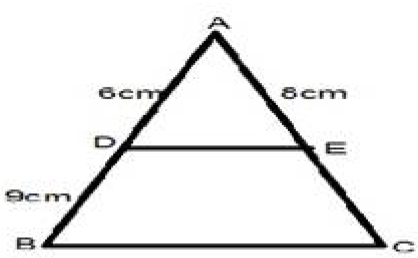
(x) If AD = 8x - 7, DB = 5x − 3, AE = 4x - 3 and EC = (3x − 1), find the value of x.

(xi) If AD = 4x − 3, AE = 8x − 7, BD = 3x − 1 and CE = 5x - 3, find the volume x.

(xii) If AD = 2.5 cm, BD = 3.0 cm and AE = 3.75 cm, find the length of AC.

Answer

(i)



we have

DE||BC

Therefore by basic proportionally theorem

AD/DB=AE/EC

6/9=8/EC

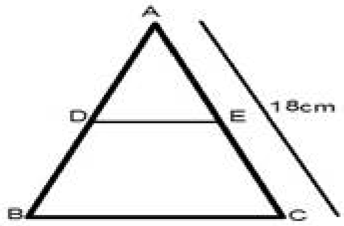
2/3=8/EC

EC=3x8/2

EC=3x4

EC=12 cm

(ii)



we have

DE||BC

Therefore by basic proportionally theorem

AD/DB=AE/EC

Adding 1 both side

AD/DB +1=AE/EC +1

3/4 +1=AE+BC/BC

3+4/4=AC/EC [AE+EC=AC]

7/4= 15/EC

EC=15x4/7

EC=60/7

Now AE+EC=AC

AE+60/7=15

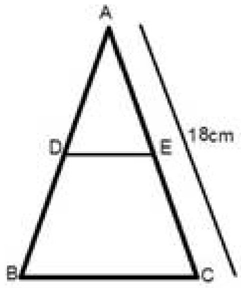
AE=15-60/7

AE=105-60/7

AE=45/7

AE=6.43 cm

(iii)



we have

DE||BC

Therefore by basic proportionally theorem

AD/DB=AE/EC

Adding 1 both side

AD/DB +1=AE/EC +1

$\frac{3}{2}$ +1=$\frac{EC}{AE}$ +1

$\frac{3+2}{2} = \frac{EC+AE}{AE}$

$\frac{5}{2}$ = AC/AE [AE+EC=AC]

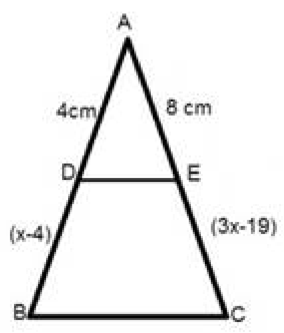
5/2=18/AE

AE= $\frac{18x2}{5}$

AE=36/5

AE=7.2 cm

(iv)



we have

DE||BC

Therefore by basic proportionally theorem

AD/DB=AE/EC

$\frac{4}{x-4} = \frac{8}{3x-19}$

4(3x-19)=8(x-4)

12x-76=8x-32

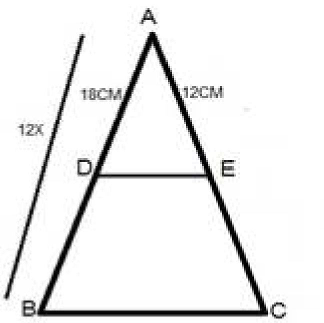
12x-8x=76-32

4x=44

x=44/4

x=11 cm

(v)



AD=8cm,AB=12cm

since BD=AB-AC

BD=12-8

BD=4 cm

DE||BC

Therefore by basic proportionally theorem

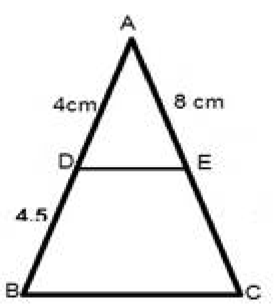
AD/DB=AE/EC

8/4=12/EC

EC=$\frac{12x4}{8}$

EC =6 cm

(vi)



we have

DE||BC

Therefore by basic proportionally theorem

AD/DB=AE/EC

4/4.5=8/EC

EC=$\frac{8x4.5}{4}$

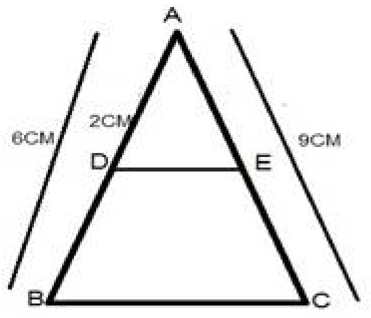
EC=9cm

Now AE+EC=AC

AC=8+9

AC=17 cm

(vii)



AD=2cm, AB=6cm

Since BD=AB-AC

BD=6-2

BD=4 cm

DE||BC

Therefore by basic proportionally theorem

AD/DB=AE/EC

Taking reciprocal on both side

DB/AD=EC/AE

4/2=EC/AE

Adding 1 both side

AD/DB +1=AE/EC +1

$\frac{4}{2}$ +1=$\frac{EC}{AE}$ +1

$\frac{4+2}{2} = \frac{EC+AE}{AE}$

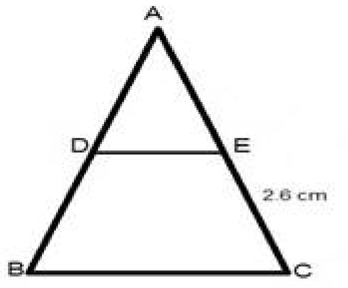
$\frac{6}{2}$ = AC/AE [AE+EC=AC]

3=9/AE

AE= $\frac{9}{3}$

AE=3 cm

(viii) we have



DE||BC

Therefore by basic proportionally theorem

AD/DB=AE/EC

4/5=AE/2.5

AE=4x2.5/5

AE=10/5

AE=2 cm

(ix) we have

DE||BC

Therefore by basic proportionally theorem

AD/DB=AE/EC

$\frac{x}{x-2} = \frac{x+2}{x-1}$

x(x-1)=(x+2)(x-2)

x^2-x=x^2-2^2

-x=-4

x=4 cm

(x) we have

DE||BC

Therefore by basic proportionally theorem

AD/DB=AE/EC

$\frac{8x-7}{5x-3} = \frac{4x-3}{3x-1}$

(8x-7)(3x-1)=(4x-3)(5x-3)

$8x(3x-1)-7(3x-1)=4x(5x-3)-3(5x-3)$

$24x^2-8x-21x+7=20x^2-12x-15x+9$

$24x^2-20x^2-29x+27x+7-9=0$

$4x^2-2x-2=0$

$2[2x^2-x-1]=0$

$2x^2-x-1=0$

$2x^2-2x-x-1=0$

$2x(x-1)+1(x-1)=0$

$(x-1)(2x+1)=0$

$x-1=0$

$x=1$

or $2x+1=0$

or $x=-1/2$

$-1/2$ is not possible.

So $x=1$

(xi) we have

$DE \parallel BC$

Therefore by basic proportionally theorem

$AD/DB=AE/EC$

$\frac{4x-3}{3x-1}=\frac{8x-7}{5x-3}$

$(8x-7)(3x-1)=(4x-3)(5x-3)$

$24x^2-8x-21x+7=20x^2-12x-15x+9$

$24x^2-20x^2-29x+27x+7-9=0$

$4x^2-2x-2=0$

$2[2x^2-x-1]=0$

$2x^2-x-1=0$

$2x^2-2x-x-1=0$

$2x(x-1)+1(x-1)=0$

$(x-1)(2x+1)=0$

x-1=0

x=1

or 2x+1=0

or x=-1/2

-1/2 is not possible.

So x=1

(xii) we have

DE||BC

Therefore by basic proportionally theorem

AD/DB=AE/EC

2.5/3=3.75/EC

EC=3.75x3/2.5

EC=375x3/250

EC=15x3/10

EC=9/2

EC=4.5 cm

Now AC=AE+EC

AC=3.75+4.5

AC=8.25 cm

2. Question

In a $\Delta\ ABC$, D and E are points on the sides AB and AC respectively. For each of the following cases show that $DE \parallel BC$:

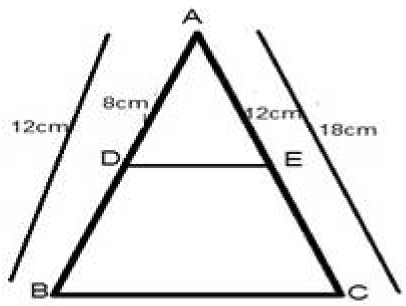
(i) AB = 12 cm, AD = 8 cm, AE = 12 cm and AC = 18 cm.

(ii) AB = 5.6 cm, AD = 1.4 cm, AE = 7.2 cm and AC = 1.8 cm.

(iii) AB = 10.8 cm, BD = 4.5 cm, AC = 4.8 cm and AE = 2.8 cm.

(iv) AD = 5.7 cm, BD = 9.5 cm, AE = 3.3 cm and EC = 5.5 cm.

Answer



(i) AB = 12 cm, AD = 8 cm, and AC = 18 cm.

∴ DB=AB-AD

= 12-8

=4 cm

EC=AC-AE

= 18-12

= 6 cm

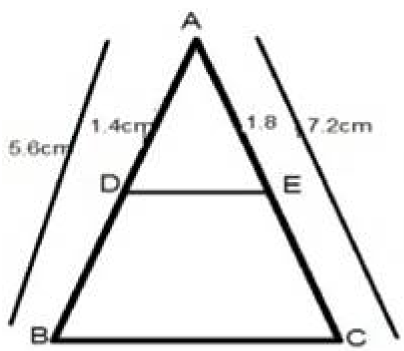
Now AD/DB=8/4=2

AE/EC=12/6=2

Thus DE divides side AB and AC of ⊿ ABC in same ratio

Then by the converse of basic proportionality theorem.

(ii)



AB = 5.6 cm, AD = 1.4 cm, AE = 1.8 cm and AC = 7.2 cm

∴ DB=AB-AD

DB=5.6-1.4

DB= 4.2 cm

And EC=AC-AE

EC= 7.2-1.8

EC=5.4

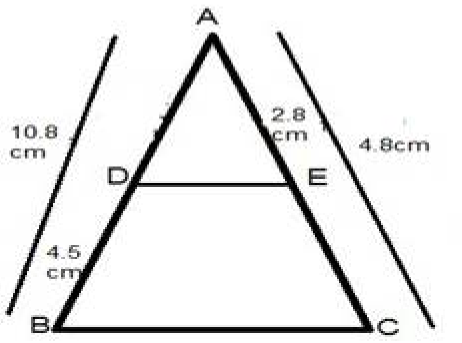
Now AD/DB=1.4/4.2=1/3

AE/EC=1.8/5.4=1/3

Thus DE divides side AB and AC of ⊿ ABC in same ratio

Then by the converse of basic proportionality theorem.

(iii)



we have

AB = 10.8 cm, BD = 4.5 cm, AC = 4.8 cm and AE = 2.8 cm

∴ AD=AB-DB

AD=10.8-4.5

AD= 6.3 cm

And EC=AC-AE

EC= 4.8-2.8

EC=2 cm

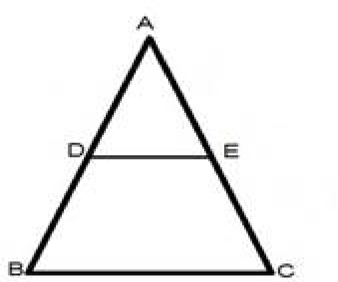
Now AD/DB=6.3/4.5=7/5

AE/EC=2.8/2=28/20=7/5

Thus DE divides side AB and AC of ⊿ ABC in same ratio

Then by the converse of basic proportionality theorem.

(iv)



DE∥BC

We have,

AD = 5.7 cm, BD = 9.5 cm, AE = 3.3 cm and EC = 5.5 cm

Now AD/DB=5.7/9.5=57/95 =3/5

AE/EC=3.3/5.5=33/55=3/5

Thus DE divides side AB and AC of ⊿ ABC in same ratio

Then by the converse of basic proportionality theorem.

3. Question

In a $\triangle ABC$, P and Q are points on sides AB and AC respectively, such that $PQ \parallel BC$. If AP = 2.4 cm, AQ = 2 cm, QC = 3 cm and BC = 6 cm, find AB and PQ.

Answer

WE have,

PQ∥BC

We have AP/PB=AQ/QC

2.4/PB=2/3

PB=3x2.4/2

PB=3x1.2

PB=3.6 cm

Now AB=AP+PB

AB=2.4+3.6

AB=6 cm

Now IN ⊿ APQ and ⊿ ABC

∠A=∠A [Common]

∠APQ=∠ABC [PQ∥BC]

⊿ APQ ~⊿ ABC [By AA criteria]

AB/AP=BC/PQ

PQ=6x2.4/6

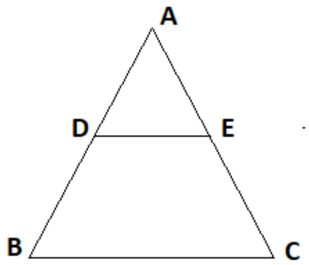
PQ=2.4 cm

4. Question

In a ΔABC, D and E are points on AB and AC respectively such that DE||BC. If AD = 2.4 cm, AE = 3.2 cm, DE = 2 cm and BC = 5 cm, find BD and CE.

Answer

In the figure given below,



Given: AD = 2.4 cm, AE = 3.2 cm, DE = 2 cm and BC = 5 cm

Let BD be x cm and CE be y cm,

Then, from, ΔADE and ΔABC, DE || BC, so by basic proportionality theorem we can write,

$$\frac{AD}{AB} = \frac{DE}{BC}$$

Or $\frac{AD}{AD + BD} = \frac{DE}{BC}$

or $\frac{2.4}{2.4 + x} = \frac{2}{5}$

or 12 = 4.8 + 2x

or x = 7.2/2

or x = DB = 3.6cm

Similarly, from ΔADE and ΔABC, we can write,

$$\frac{AE}{AC} = \frac{DE}{BC}$$

Or $\frac{AE}{AE + EC} = \frac{DE}{BC}$

or $\frac{3.2}{3.2 + y} = \frac{2}{5}$

or 16 = 6.4 +2y

or y = 9.6/2

or y = CE = 4.8 cm

Thus, the lengths of BD and CE are 3.6 cm and 4.8 cm respectively.

5. Question

In Fig. 4.35, state if $PQ \parallel EF$.

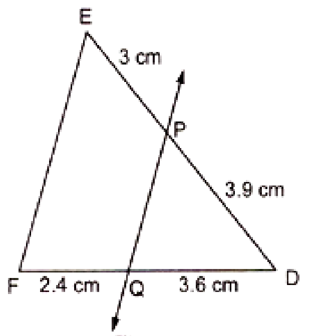


Fig. 4.35

Answer

DP/PE=3.9/3=1.3/1=13/10

DQ/QF=3.6/2.4=36/24=3/2

DP/PE≠DQ/QF

So PQ is not parallel to EF

6. Question

M and N are points on the sides PQ and PR respectively of a ΔPQR. For each of the following cases, state whether $MN \parallel QR$:

(i) PM = 4 cm, QM = 4.5 cm, PN = 4 cm, NR = 4.5 cm

(ii) PQ = 1.28 cm, PR = 2.56 cm, PM = 0.16 cm, PN = 0.32 cm

Answer

(i) we have PM=4cm, QM=4.5 cm,PN=4 cm and NR=4.5 cm

Hence PM/QM=4/4.5=40/45=8/9

PN/NR=4/4.5=40/45=8/9

PM/QM= PN/NR

by the converse of proportionality theorem

MN∥QR

(ii) we have PQ=1.28cm, PR=2.56 cm,PM=0.16 cm and PN=0.32 cm

Hence PQ/PR=1.28/2.56=128/256=1/2

PM/PN=0.16/0.32=16/32=1/2

PQ/PR = PM/PN

by the converse of proportionality theorem

MN∥QR

7. Question

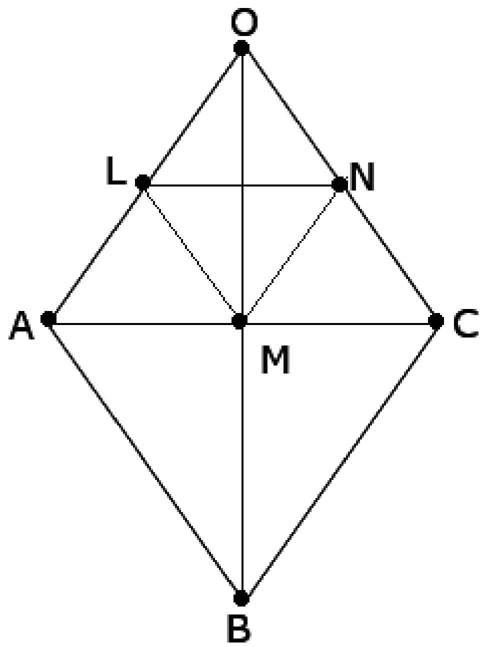
In three line segments OA, OB, and OC, points L, M, N respectively are so chosen that $LM \parallel AB$ and $MN \parallel BC$ but neither of L, M, N nor of A, B, C are collinear. Show that $LN \parallel AC$.

Answer

Given: In three line segments OA, OB, and OC, points L, M, N respectively are so chosen that $LM \parallel AB$ and $MN \parallel BC$ but neither of L, M, N nor of A, B, C are collinear.

To show : $LN \parallel AC$

Solution:



We have LM∥AB and MN∥BC

by the basic proportionality theorem

OL/AL=OM/MB ....(i)

ON/NC=OM/MB ....(ii)

Comparing equ.(i)and(ii)

OL/AL=ON/NC

Thus LN divides side OA and OC of ⊿ OAC in same ratio

Then by the converse of basic proportionality theorem

$LN \parallel AC$

8. Question

If D and E are points on sides AB and AC respectively of a ΔABC such that $DE \parallel BC$ and BD = CE. Prove that ΔABC is isosceles.

Answer

We have DE∥BC

by the converse of proportionality theorem

AD/DB=AE/EC

AD/DB=AE/DB [BD=CE]

AD=AE

Adding D both sides

AD+BD=AE+DB

AD+BD=AE+EC [BD=CE]

AB=AC

⊿ABC is isosceles

Exercise 4.3

1. Question

In a ΔABC, AD is the bisector of $\angle A$, meeting side BC at D.

(i) If BD = 2.5 cm, AB = 5 cm and AV = 4.2 cm, find DC.

(ii) If BD = 2 cm, AB = 5 cm and DC = 3 cm, find AC.

(iii) If AB = 3.5 cm, AC = 4.2 cm and DC = 2.8 cm, find BD.

(iv) If AB = 10 cm, AC = 14 cm and BC = 6 cm, find BD and DC.

(v) If AC = 4.2 cm, DC = 6 cm and BC = 10 cm, find AB.

(vi) If AB = 5.6 cm, AC = 6 cm and DC = 6 cm, find BC.

(vii) If AD = 5.6 cm, BC = 6 cm and BD = 3.2 cm, find AC.

(viii) If AB = 10 cm, AC = 6 cm and BC = 12 cm, find BD and DC.

Answer

(i) we have

Angle BAD=CAD

Here AD bisects ∠A

BD/DC=AB/AC

2.5/DC=5/4.2

DC=2.5*4.2/5

DC=2.1 cm

(ii) Here AD bisects ∠A

AB/DC=AB/AC

2/3=5/AC

AC=15/2

AC=7.5 cm

(iii) in △ ABC A bisects ∠A

BD/DC=AB/BC

BD/2.8=3.5/4.2

BD=3.5*2.8/4.2

BD=7/3

BD=2.33 cm

(iv) In△ABC, AD bisects ∠A

BD/DC=AB/AC

X/6-x =10/14

14x=60-10x

14x+10x=60

24x=60

x= 60/24

x=5/2

x=2.5

BD=2.5

DC= 6-2.5

DC=3.5

(v) AB/AC=BD/DC

AB/4.2=BC-DC/DC

AB/4.2=10-6/6

AB/4.2=4/6

AB=4*4.2/6

AB=2.8 cm

(vi) BD/DC=AB/AC

BD/6=5.6/6

BD=5.6

BC= BD+DC

BC=5.6+6

BC=11.6 cm

(viii) In△ABC, AD bisects ∠A

AB/AC=BD/DC

5.6/AC=3.2/BC-BD

5.6/AC=3.2/6-3.2

5.6/AC=3.2/2.8

AC*3.2=2.8*5.6

AC=2.8*5.6/3.2

AC=7*0.7

AC=4.9 cm

(ix) let BD=x,then DC=12-X

BD/DC=AB/BC

x/12-x= 10/6

6x=120-10x

6x+10x=120

16x=120

x=120/16

x= 7.5

BD=7.5 cm

DC =12-x

DC=12-7.5

DC=4.5 cm

2. Question

In Fig. 4.57, AE is the bisector of the exterior $\angle CAD$ meeting BC produced in E. If AB = 10 cm, AC = 6 cm and BC = 12 cm, find CE.

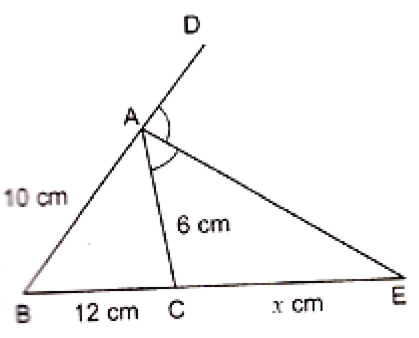


Fig. 4.57

Answer

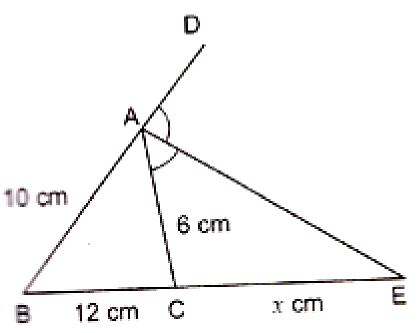


Fig. 4.57

AE is the bisector of ∠A

We know that external bisector of an angle of a triangle divides the opposite side externally in the ratio of the sides containing the angles.

$$\frac{BE}{CE} = \frac{AB}{AC}$$

$$\Rightarrow \frac{12+x}{x} = \frac{10}{6}$$

⇒ 10X=6(12+x)

⇒ 10X=72+6X

⇒ 10X-6X=72

⇒ 4X =72

⇒ x=72/4

⇒ x=18

3. Question

In Fig. 4.58, ΔABC is a triangle such that $\frac{AB}{AC} = \frac{BD}{DC}$, $\angle B = 70^0$, $\angle C = 50^0$. Find $\angle BAD$.

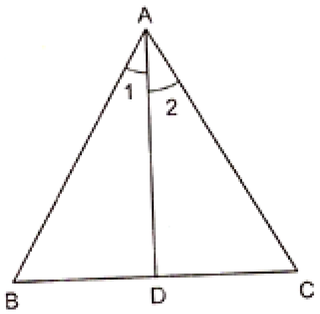


Fig. 4.58

Answer

We have

AB/AC=BD/DC

∴ ∠1=∠2

IN ⊿ABC

∠A+∠B+∠C=180

∠A+70+50=180

∠A+120=180

∠A=180-120

∠A=60

∠1+∠2=60 (∠1+∠2=∠A)

∠1+∠1=60 (∠1=∠2)

2∠1=60

∠1=60/2

∠1=30

∠BAD=30

4. Question

In ΔABC (fig. 4.59), if $\angle 1 = \angle 2$, prove that $\frac{AB}{AC} = \frac{BD}{DC}$.

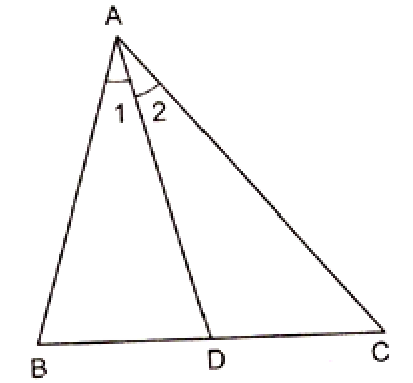


Fig. 4.59

Answer

∠1=∠2 (Given)

Draw a line EC∥AD

AC bisects them

∴∠2=∠3 (by alternate angle) ………………. (i)

∠1=∠4 (corresponding angle) …………….(ii)

∠1=∠2 (given)

From equ (i) and equ (ii)

∠3=∠4

or AE=AC …………….(III)

Now ,⊿ BCE

BD/DC=BA/AE (BY PROPORTIONALITY THEORAM)

BD/DC=AB/AC (∵ BA=AB AND AE=AC from equ (iii))

Hence AB/AC=BA/DC Proved

5. Question

D, E and F are the points on sides BC, CA and AB respectively of $\triangle ABC$ such that AD bisects $\angle A$, BE bisects $\angle B$ and CF bisects $\angle C$. If AB = 5 cm, BC = 8 cm and CA = 4 cm, determine AF, CE and BD.

Answer

in⊿ ABC

CF bisects ∠A

∴ AF/FB=AE/AC

AF/5-AF=4/8

2AF=5-AF

2AF+AF=5

3AF=5

AF=5/3 cm

⊿ABC, BE bisects ∠B

∴ AE/AC=AB/BC

4-CE/CE=5/8

5CE=32-8CE

5CE+8CE=32

13CE=32

CE=32/13 cm

Similarly

BD/DC=AB/AC

BD/8-BD=5/4

4BD=40-5BD

4BD+5BD=40

9BD=40

BD=40/9 cm

6. Question

In Fig. 4.60, check whether AD is the bisector of $\angle A$ of ΔABC in each of the following:

(i) AB = 5 cm, AC = 10 cm, BD = 1.5 cm and CD = 3.5 cm

(ii) AB = 4 cm, AC = 6 cm, BD = 1.6 cm and CD = 2.4 cm

(iii) AB = 8 cm, AC = 24 cm, BD = 6 cm and BC = 24 cm

(iv) AB = 6 cm, AC = 8 cm, BD = 1.5 cm and CD = 2 cm

(v) AB = 5 cm, AC = 12 cm, BD = 2.5 cm and BC = 9 cm

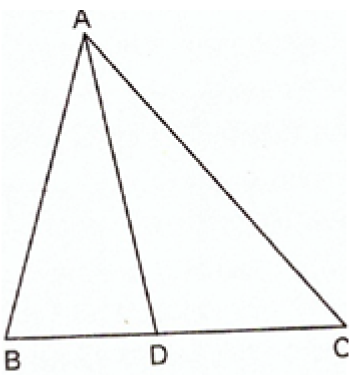


Fig. 4.60

Answer

(i) BD/DC=AB/AC

1.5/3.5=5/10

15/35*10/10=1/2

3/7=1/2

Not bisects

(ii) 1.6/2.4=4/6

16/24=2/3

2/3=2/3

bisects

(iii) BD/CD=AB/AC

BD/BC-BD=AB/AC

BD/24-6=8/24

6/18=1/3

1/3=1/3

bisects

(iv) 1.5/2= 6/8

3/4=3/4

bisects

(v) BD/CD=AB/AC

BD/BC-BD=AB/AC

BD/9-2.5=5/12

2.5/6.5=5/12

5/13=5/12

Not bisects

7. Question

In Fig. 4.60, AD bisects $\angle A$, AB = 12 cm, AC = 20 cm and BD = 5 cm, determine CD.

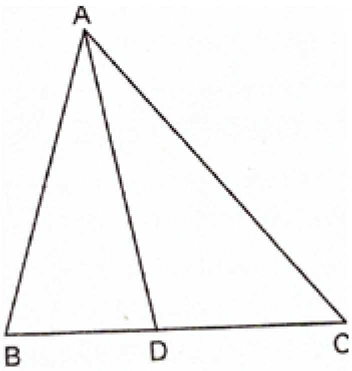


Fig. 4.60

Answer

AD bisects ∠A

∴ AB/AC=BD/CD

12/20=5/CD

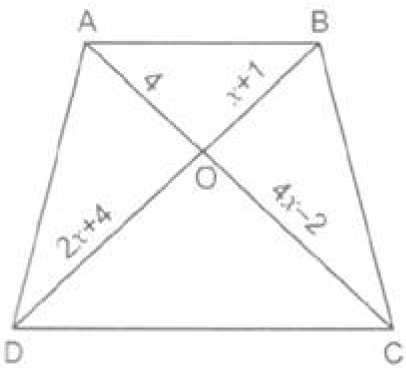
CD =100/12

CD=8.33 cm

Exercise 4.4

1 A. Question

(i) In fig. 4.70, if $AB \parallel CD$, find the value of x.



Answer

Diagonal of trapezium divide each other proportiona

AO/OC=BO/OD

4/4X-2=x+1/2x+4

$4x^2$-2x+4x-2=8x+16

$4x^2$+2x-2-8x-16=0

$4x^2$-6x-18=0

2($2x^2$-3x-9)=0

$2x^2-3x-9=0$

$2x^2-6x+3x-9=0$

2x(x-3)+3(x-3)=0

(x-3)(2x+3)=0

x-3=0

x=3

or,2x+3=0

2x=-3

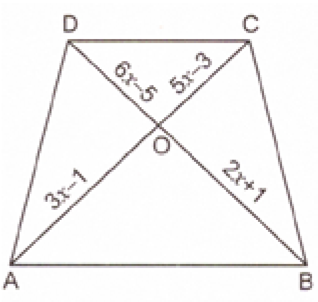
x= -3/2

x=-3/2 is not possible

So x=3

1 B. Question

In Fig. 4.71, if $AB \parallel CD$, find the value of x.



Answer

AO/OC=BO/OD

3x-1/5x-3=2x+1/6x-5

(3x-1)(6x-5)= (2x+1) (5x-3)

$18x^2-15x-6x+5=10x^2-6x+5x-3$

$18x^2-21x+5=10x^2-x-3$

$18x^2-21x+5-10x^2+x+3=0$

$8x^2-20x+8=0$

$4(2x^2-5x+2)=0$

$2x^2-5x+2=0$

$2x^2-4x-x+2=0$

2x(x-2)-1(x-2)=0

(x-2)(2x-1)=0

x-2=0

x=2

Or, 2x-1=0

2x=1

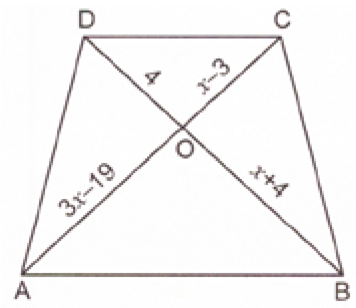
x=1/2

But x=1/2 is not possible

So x=2

1 C. Question

In Fig. 4.72, $AB \parallel CD$. If OA = 3x − 19, OB = x − 4, OC = x − 3 and OD = 4, find x.



Answer

AO/OC=BO/OD

3X-19/X-3=X-4/4

(x-3)(x-4)=4(3x-19)

X^2 -4x-3x+12=12x-76

X^2 -7x+12-12x+76=0

X^2 -19x+88=0

X^2 -11x-8x+88=0

X(x-11)-8(x-11)=0

(x-11)(x-8)=0

x-11=0

x=11

or x-8=0

x=8

x=11 or 8

Exercise 4.5

1. Question

In Fig. 4.136, $\triangle ABC \sim \triangle APQ$. If BC = 8 cm, PQ = 4 cm, BA = 6.5 cm and AP = 2.8 cm, find CA and AQ

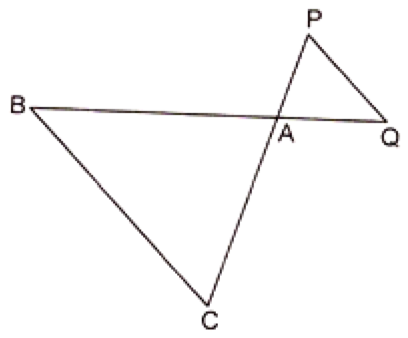


Fig. 4.136

Answer

Given ΔACB ~ ΔAPQ

Then, AC/AP = BC/PQ = AB/AQ

Or AC/2.8 = 8/4 = 6.5/AQ

Or AC/2.8 = 8/4 and 8/4 = 6.5/AQ

Or AC = 8/4 x 2.8 and AQ = 6.5 x4/8

Or AC=5.6cm and AQ = 3.25cm

2. Question

A vertical stick 10 cm long casts a shadow 8 cm long. At the same time a tower casts a shadow 30 m long. Determine the height of the tower.

Answer

Length of stick = 10cm

Length of shadow stick= 8cm

Length of shadow of tower = hcm

In ΔABC and ΔPQR

<B = <C = 90° And <C = <R (Angular elevation of sum)

Then ΔABC ~ ΔPQR (By AA similarty)

So, $\frac{AB}{PQ} = \frac{BC}{QR}$

Or $\frac{10cm}{8cm} = \frac{H}{3000}$

Or h= $\frac{10}{8}$ x 3000

Or 3750cm

Or 37.5m

3. Question

In Fig. 4.137, $AB \parallel QR$. Find the length of PB.

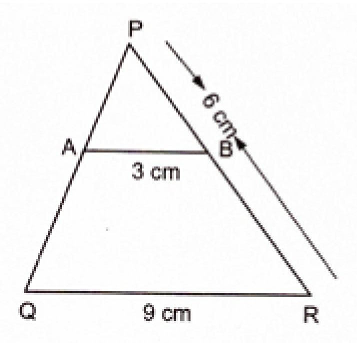


Fig. 4.137

Answer

We have ΔPAB and ΔPQR

<P = <P (Common)

<PAB = <PQR (Corresponding angles)

Then, ΔPAB ~ ΔPQR (BY AA similarity)

So, $\frac{PB}{PR} = \frac{AB}{QR}$ (Corresponding parts of similar triangle area proportion)

Or , $\frac{PB}{6} = \frac{3}{9}$

Or PB = $\frac{3}{9}$ x 6

Or PB= 2cm

4. Question

In Fig. 4.138, $XY \parallel BC$. Find the length of XY.

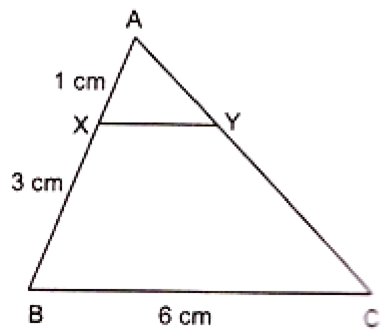


Fig. 4.138

Answer

We have , XY||BC

In Δ AXY and ΔABC

<A = <A (Common)

<AXY = <ABC (Corresponding angles)

Then, Δ AXY ~ΔABC (By AA Similarity)

So, $\frac{AX}{BY} = \frac{XY}{BC}$ (Corresponding parts of similar triangle area proportion)

Or $\frac{1}{4} = \frac{XY}{6}$

Or XY = 6/4

Or XY = 1.5cm

5. Question

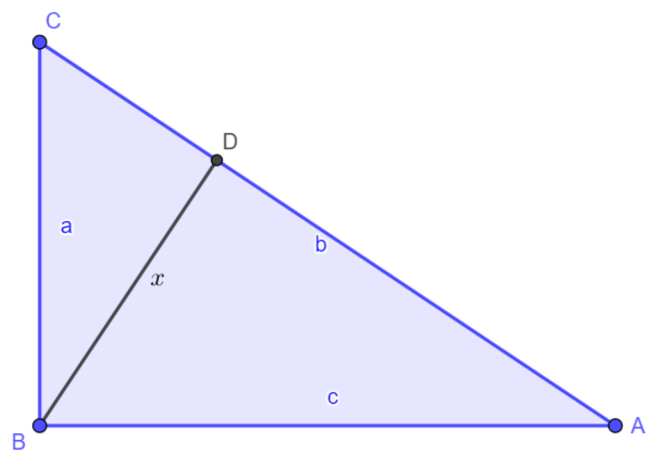
In a right angled triangle with sides a and b and hypotenuse c, the altitude drawn on the hypotenuse is x. Prove that ab = cx

Answer

Given: In a right angled triangle with sides a and b and hypotenuse c, the altitude drawn on the hypotenuse is x.

To prove: ab = cx

Proof: Let in a right-angled triangle ABC at B, a perpendicular from C to AB is drawn such thatBC = aAC = bBA = cBD = x



In ΔABC and ΔCDB

∠B = ∠B (Common)

∠ABC = ∠CDB (Both 90°)

Then, ΔABC ~ ΔCDB (By AA Similarity)

So, $\frac{AC}{CD} = \frac{AB}{CB}$ (Corresponding parts of similar triangle area proportion)

Or $\frac{b}{x} = \frac{c}{a}$

Or ab = cx

6. Question

In Fig. 4.139, $\angle ABC = 90°$ and $BD \perp AC$. If BD = 8 cm and AD = 4 cm, find CD.

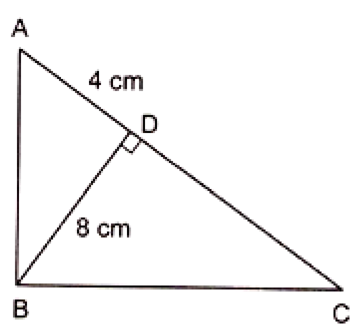


Fig. 4.139

Answer

We have, <ABC = 90° and BD perpendicular AC

Now, <ABD + <DBC – 90° ………..(I) (<ABC – 90°)

And <C + <DBC – 90° ………..(II) (By angle sum Prop. in ΔBCD) Compare equation I &II

<ABD = <C ………..(III)

In ΔABD and ΔBCD

<ABD = <C (From equation I)

<ADB = <BDC (Each 90°)

Then, ΔABD ~ΔBCD (By AA similarity)

So, $\frac{BD}{CD} = \frac{AD}{BD}$ (Corresponding parts of similar triangle area proportion)

Or, $\frac{8}{CD} = \frac{4}{8}$

Or CD = $\frac{8 X 8}{4}$

Or CD = 16cm

7. Question

In Fig. 4.140, $\angle ABC$ = 90° and $BD \perp AC$. If AB = 5.7 cm, BD = 3.8 cm and CD = 5.4 cm, find BC.

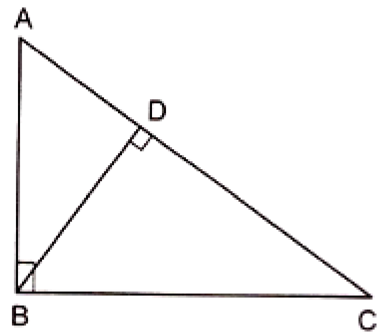


Fig. 4.140

Answer

We have , <ABC = 90° and BD Perpendicular AC

In Δ ABY and ΔBDC

<C = <C (Common)

<ABC = <BDC (Each 90° angles)

Then, Δ ABC ~ΔBDC (By AA Similarity)

So, $\frac{AB}{BD} = \frac{BC}{DC}$ (Corresponding parts of similar triangle area proportion)

Or $\frac{5.7}{3.8} = \frac{BC}{5.4}$

Or BC = 5.7/3.8 x 8.1

Or BC = 12.15cm

8. Question

In Fig. 4.141 $DE \parallel BC$ such that AE = (1/4) AC. If AB = 6 cm, find AD.

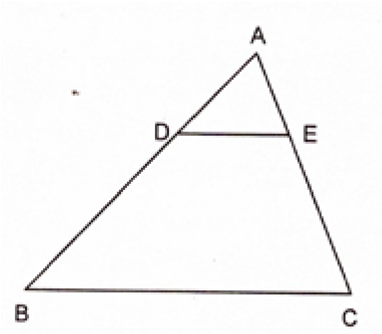


Fig. 4.141

Answer

We have, DE||BC, AB = 6cm and AE = 1/4 AC

In ΔADE and ΔABC

<A = <A (Common)

<ADE = <ABC (Corresponding angles)

Then, ΔADE ~ ΔABC (By AA similarity)

So, $\frac{AD}{AB} = \frac{AE}{AC}$ (Corresponding parts of similar triangle area proportion)

Or $\frac{AD}{6} = \frac{\frac{1}{4}AC}{AC}$ (AE = 1/4 AC Given)

Or , $\frac{AD}{6} = \frac{1}{4}$

Or, AD = 6/4

Or, AD = 1.5cm

9. Question

In Fig. 4.142, PA, QB and RC are each perpendicular to AC. Prove that $\frac{1}{x} + \frac{1}{z} = \frac{1}{y}$.

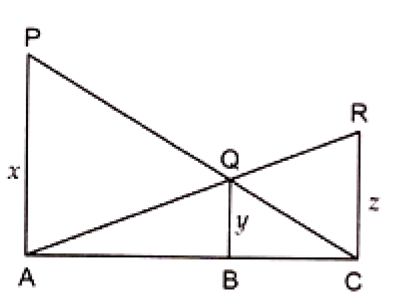


Fig. 4.142

Answer

We have, PA ⊥ AC, and RC ⊥ AC

Let AB = a and BC = b

In ΔCQB and ΔCPA

<QCB = <PCA (Common)

<QBC = <PAC (Each 90°)

Then, ΔCQB ~ ΔCPA (By AA similarity)

So, $\frac{QB}{PA} = \frac{CB}{CA}$ (Corresponding parts of similar triangle area proportion)

Or, $\frac{y}{z} = \frac{b}{a+b}$ -----------(i)

In ΔAQB and ΔARC

<QAB = <RAC (Common)

<ABQ = <ACR (Each 90°)

Then, ΔAQB ~ ΔARC (By AA similarity)

So, $\frac{QB}{RC} = \frac{AB}{CA}$ (Corresponding parts of similar triangle area proportion)

Or, $\frac{y}{x} = \frac{a}{a+b}$ -----------(ii)

Adding equation i & ii

$\frac{y}{x} + \frac{y}{z} = \frac{b}{a+b} = \frac{a}{a+b}$

Or, y $(\frac{1}{x} + \frac{1}{z}) = \frac{b+a}{a+b}$

Or, y $(\frac{1}{x} + \frac{1}{z}) = 1$

Or, $\frac{1}{x} + \frac{1}{z} = \frac{1}{y}$

10. Question

In Fig. 4.143, $\angle A = \angle CED$, prove that $\Delta\, CAB \sim \Delta\, CED$. Also, find the value of x.

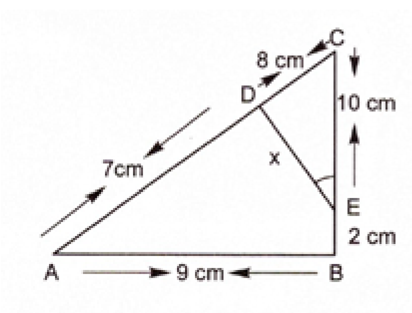


Fig.4.143

Answer

We have, <A = <CED

In ΔCAB and ΔCED

<C = <C (Common)

<A = <CED (Given)

Then, ΔCAB ~ ΔCED (By AA similarity)

So, $\frac{CA}{CE} = \frac{AB}{ED}$ (Corresponding parts of similar triangle area proportion)

Or,15/9 = 9/x

Or, 15x = 90

Or, x = 90/6

Or, x = 6cm.

11. Question

The perimeters of two similar triangles are 25 cm and 15 cm respectively. If one side of first triangle is 9 cm, what is the corresponding side of the other triangle?

Answer

Assume ABC and PQR to be 2 triangle.

We, have

ΔABC ~ΔPQR

Perimeter of ΔABC = 25cm

Perimeter of ΔPQR = 15cm

AB = 9cm

PQ = ?

Since, ΔABC ~ΔPQR

Then, ratio of perimeter of triangles = ratio of corresponding sides

So, $\frac{25}{15} = \frac{AB}{PQ}$ (Corresponding parts of similar triangle area proportion)

Or $\frac{25}{15} = \frac{9}{PQ}$

Or PQ = 135/25

Or PQ = 5.4 cm

12. Question

In $\triangle ABC \text{ and } \triangle DEF$, it is being given that: AB = 5 cm, BC = 4 cm and CA = 4.2 cm; DE = 10 cm, EF = 8 cm and FD = 8.4 cm. If $AL \perp BC$ and $DM \perp EF$, find AL : DM.

Answer

Since $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{1}{2}$

Then, ΔABC ~ ΔDEF (By SS similarity)

Now, In ΔABL ~ ΔDEM

<B = <E (ΔABC ~ΔDEF)

<ALB =<DME (Each 90°)

Then, ΔABL ~ ΔDEM (By SS similarity)

So, $\frac{AB}{DE} = \frac{AL}{DM}$ (Corresponding parts of similar triangle area proportion)

Or $\frac{5}{10} = \frac{AL}{DM}$

Or, $\frac{1}{2} = \frac{AL}{DM}$

13. Question

D and E are the points on the sides AB and AC respectively of a ΔABC such that AD = 8 cm, DB = 12 cm, AE = 6 cm and CE = 9 cm. Prove that BC = 5/2 DE.

Answer

We have ,

$\frac{AD}{DB} = \frac{8}{12} = \frac{2}{3}$

And, $\frac{AD}{EC} = \frac{6}{9} = \frac{2}{3}$

Since, $\frac{AD}{DB} = \frac{AD}{EC}$

Then , by converse of basic proportionality theorem.

DE||BC

In Δ ADE and Δ ABC

<A= <A (Common)

<ADE = <B (Corresponding angles)

Then, Δ ADE ~ Δ ABC (By AA similarity)

$\frac{AD}{AB} = \frac{DE}{BC}$ (Corresponding parts of similar triangle are proportion)

$$\frac{8}{20} = \frac{DE}{BC}$$

$$\frac{2}{5} = \frac{DE}{BC}$$

BC = 5/2 DE

14. Question

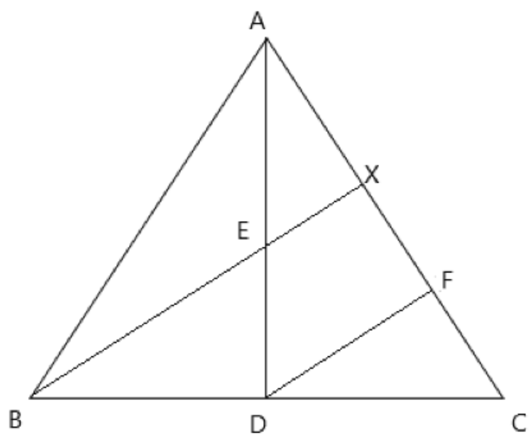
D is the mid-point of side BC of a ΔABC. AD is bisected at the point E and BE produced cuts AC at the point X. Prove that BE : EX = 3 : 1.

Answer

Given:- In ΔABC, D is the midpoint of BC and E is the midpoint of AD.

To prove:- BE: EX = 3 : 1

Proof: Const:- Through D, Draw DF||BX



In ΔEAX and Δ ADF

∠EAX = ∠DAF (Common)

∠AXE = ∠DFA (Corresponding angles)

By AA similarity,

ΔEAX ~ Δ ADF

So, $\frac{EX}{DF} = \frac{AE}{AD}$ (Corresponding parts of similar triangle are proportion)

As E is mid point of AD

$\Rightarrow \frac{EX}{DF} = \frac{AE}{2AE}$

Or, DF = 2EX. ....(i)

In ΔDCF and ΔBCX ∠DCY = ∠BCX (common)∠CFD = ∠CXB (Corresponding angles)By AA similarity,ΔDCF ~ ΔBCX

SO, $\frac{CD}{CB} = \frac{DF}{BX}$ (Corresponding parts of similar triangle area proportion)

As D is mid point of BC and E is mid point of AD.

$$\Rightarrow \frac{CD}{2CD} = \frac{DF}{BE + EX}$$

Or $\frac{1}{2} = \frac{DF}{BE+EX}$

Or BE + EX = 2DFFrom (i)

BE + EX = 4EX

⇒ BE = 4EX − EX

⇒ BE = 4EX − EX

⇒ BE = 3EX

⇒ BE/EX =3/1

⇒ BE:Ex = 3:1

15. Question

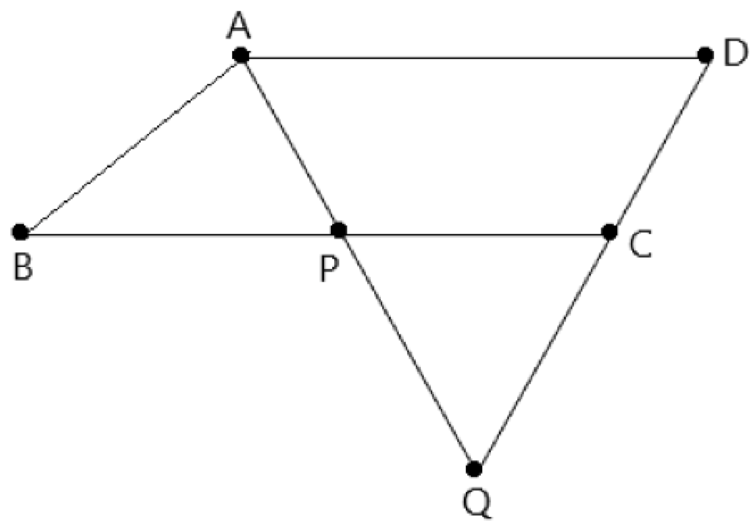
ABCD is a parallelogram and APQ is a straight line meeting BC at P and DC produced at Q. Prove that the rectangle obtained by BP and DQ is equal to the rectangle contained by AB and BC.

Answer

Given :- ABCD is a parallelogram

To prove :- BP x DQ = AB x BC

Proof:-



In ΔABP and ΔQDA

<B = <D (Opposite angles of parallelogram)

<BAP = <AQD (Alternative interior angle)

Then, ΔABP ~ ΔQDA

SO, $\frac{AB}{QD} = \frac{BP}{DA}$ (Corresponding parts of similar triangle area proportion) But, DA = BC (Opposite side of parallelogram)But DA = BC (opposite sides of parallelogram)

Then, $\frac{AB}{QD} = \frac{BP}{BC}$

Or, AB x BC = QD X BPHence proved

16. Question

In $\triangle ABC$, AL and CM are the perpendiculars from the vertices A and C to BC and AB respectively. If AL and CM intersect at O, prove that :

(i) $\triangle OMA \sim \triangle OLC$

(ii) $\frac{OA}{OC} = \frac{OM}{OL}$

Answer

We have

AL $\perp$ BC and CM $\perp$ AB

IN ΔOMA and ΔOLC

<MOA = <LOC (Vertically opposite angles)

<AMO = <LOC (Each 90°)

Then, ΔOMA ~ΔOLC (BY AA Similarity)

SO, $\frac{OA}{OC} = \frac{OM}{OL}$ (Corresponding parts of similar triangle area proportion)

17. Question

In fig. 4.144, we have AB||CD||EF. If AB = 6 cm, CD = x cm, EF = 10 cm, BD = 4 cm and DE = y cm, calculate the values of x and y.

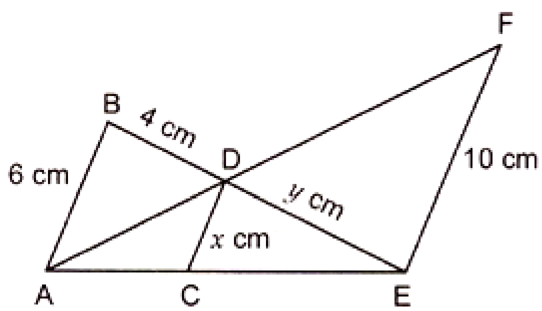


Fig. 4.144

Answer

We have AB||CD. If AB = 6cm, CD = xcm, EF = 10 cm, BD = 4cm and DE = ycm

In ΔECD and ΔEAB

<ECD = <EAB (Corresponding angles)

Then, ΔECD ~ ΔEAB ....(i) (By AA similarity)

SO, $\frac{EC}{EA} = \frac{CD}{AB}$ (Corresponding parts of similar triangle are proportion)

Or $\frac{EC}{EA} = \frac{x}{6}$ ....(ii)

In ΔACD and ΔAEF

<CAD = <EAF (Common)

<ACD = <AEF (Corresponding angles)

Then, ΔACD ~ ΔAEF (By AA similarity)

SO, $\frac{AC}{AE} = \frac{CD}{EF}$

Or, $\frac{AC}{AE} = \frac{x}{10}$........................ (iii)

Adding equation iii & ii

So, $\frac{AC}{AE} + \frac{EC}{EA} = \frac{x}{6} + \frac{x}{10}$

Or, $\frac{AE}{AE} = \frac{5x + 3x}{30}$

Or, 1 = $\frac{8x}{30}$

Or, x = $\frac{30}{8}$

Or, x = 3.75cm

From (i)$\frac{DC}{AB} = \frac{CD}{BE}$

Or,$\frac{3.75}{6} = \frac{y}{y+4}$

Or, 6y = 3.75y +15

Or, 2.25y = 15

Or, y = $\frac{15}{2.25}$

Or, y = 6.67cm

18. Question

ABCD is a quadrilateral in which AD = BC. If P, Q, R, S be the mid-points of AB, AC, CD and BD respectively, show that PQRS is a rhombus.

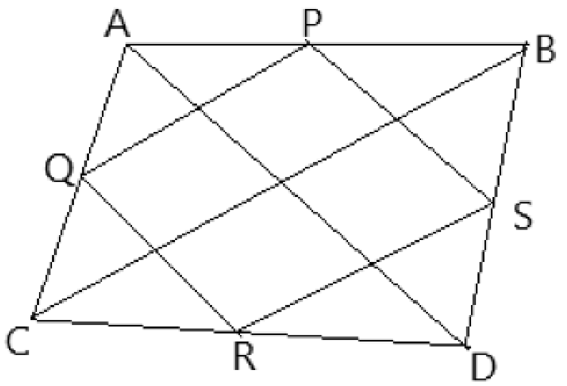
Answer

Given: ABCD is a quadrilateral in which AD = BC. P, Q, R, S be the mid-points of AB, AC, CD and BD respectively.

To show: PQRS is a rhombus.

Solution: So, we have, a quadrilateral ABCD where AD = BC

And P, Q, R and S are the mid-point of the sides AB, AC, and BD.



We need to prove that PQRS is a rhombus.

In ΔBAD, P and S are the mid points of the sides AB and BD respectively,By midpoint theorem which states that the line joining mid-points of a triangle is parallel to third side we get,

PS||AD and PS = 1/2 AD…………(i)

In ΔCAD, Q and R are the mid points of the sides CA and CD respectively,by midpoint theorem we get,

QR||AD and QR = 1/2 AD …………..(ii)

Compare (i) and (ii)

PS||QR and PS = QR

Since one pair of opposite sides is equal and parallel,

Then, we can say that PQRS is a parallelogram…………(iii)

Now, In ΔABC,P and Q are the mid points of the sides AB and AC respectively,by midpoint theorem,

PQ||BC and PQ = 1/2 BC…………..(iv)

And AD = BC …………………………..(v) (given)

Compare equations (i) (iv) and (v), we get,

PS = PQ ………………………………….(vi)

From (iii) and (vi), we get,

PS = QR = PQ Therefore, PQRS is a rhombus.

19. Question

In Fig. 4.145, If $AB \perp BC$, $DC \perp BC$ and $DE \perp AC$, prove that $\Delta CED \sim \Delta ABC$.

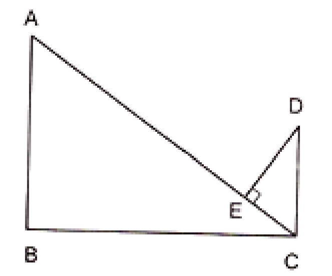


Fig. 4.145

Answer

Given AB⊥BC, DC ⊥ BC and DE ⊥AC

To prove:- ΔCED ~ΔABC

Proof:-

<BAC + <BCA = 90° …………..(i) (By angle sum property)

And, <BCA + <ECD = 90°……(ii) (DC ⊥ BC given)

Compare equation (i) and (ii)

<BAC = <ECD……………..(iii)

In ΔCED and ΔABC

<CED = <ABC (Each 90°)

<ECD = <BAC (From equation iii)

Then, ΔCED ~ΔABC.

20. Question

In an isosceles $\Delta\, ABC$, the base AB is produced both the ways to P and Q such that AP × BQ = AC2. Prove that $\Delta\, APC \sim \Delta\, BCQ$.

Answer

Given : In ΔABC , CA – CB and AP x BQ = AC^2

To prove :- ΔAPC ~ BCQ

Proof:-

AP X BQ = AC^2 (Given)

Or, AP x BC = AC x AC

Or, AP x BC = AC x BC (AC = BC given)

Or, AP/BC = AC/PQ ………………(i)

Since, CA = CB (Given)

Then, <CAB = <CBA …………….(ii) (Opposite angle to equal sides)

NOW, <CAB +<CAP = 180° ....(iii) (Linear pair of angle)

And <CBA + <CBQ = 180° ....(iv) (Linear pair of angle)

Compare equation (ii) (iii) & (iv)

<CAP = <CBQ................(v)

In ΔAPC and ΔBCQ

<CAP = < CBQ (From equation v)

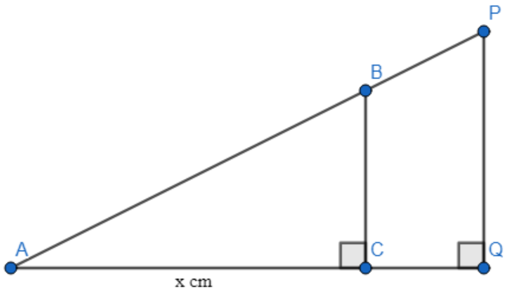
AP/BC = AC/PQ (From equation i)

Then , ΔAPC ~ ΔBCQ (By SAS similarity)

21. Question

A girl of height 90 cm is walking away from the base of a lamp-post at a speed of 1.2 m/sec. If the lamp is 3.6 m above the ground, find the length of her shadow after 4 seconds.

Answer



We have,

Let P be a lamb at a height of 3.6 m above that ground i.e. PQ = 3.6 m

Let BC be a girl, such that CQ is distance she covered and Let AC be her shadow, Height of girl = AB = 90cm = 0.9m

Height of lamp post = PQ = 3.6m

Speed of girl = 1.2 m/sec

So, Distance moved by the girl(CQ) = Speed x time

= 1.2 x 4 = 4.8m

Let length of shadow (AC) = 'x' cm

Then, AQ = AC + CQ = x + 4.8

In ΔABC and ΔAPQ

∠ACB = ∠AQP (Each 90 °)

∠BAC = ∠PAQ (Common)

Then , ΔABC ~ ΔAPQ (By AA similarity)

So, AC/AQ = BC/ PQ(Corresponding parts of similar triangle are proportional)

Or, x/x +4.8 = 0.9/3.6

Or, x/x +4.8 = 1/4

Or, 4x = x + 4.8

Or, 4x − x = 4.8

Or, 3x = 4.8

Or x = 4.8/3

Or x = 1.6mi.e. length of shadow is 1.6 m.

22. Question

Diagonals AC and BD of a trapezium ABCD with $AB \parallel DC$ intersect each other at the point O. Using similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$.

Answer

We have,

ABCD is a trapezium with AB || DC

In ΔAOB and ΔCOD <AOB = <COD (Vertically opposite angle)

<OAB = <OCD (Alternate interior angle)

Then, ΔAOB ~ΔCOD (By AA similarity)

So, OA/OC = OB/OD(Corresponding parts of similar triangle are proportional)

23. Question

If ΔABC and ΔAMP are two right triangles, right angled at B and M respectively such that $\angle MAP = \angle BAC$. Prove that

(i) $\Delta ABC \sim \Delta AMP$

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$

Answer

We have,

<B = <M = 90°

And, <BAC = <MAP

In ΔABC and ΔAMP

<B = <M (each 90°)

<BAC = <MAP (Given)

Then, ΔABC ~ΔAMP (By AA similarity)

So, CA/PM = BC/MP(Corresponding parts of similar triangle are proportional)

24. Question

A vertical stick of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Answer

Let AB be a tower

CD be a stick , CD = 6m

Shadow of AB is BE = 28cm

Shadow of CD is DF = 4m

At same time light rays from sun will fail on tower and stick at the same angle

So, <DCF = <BAE

And <DFC = <BEA

< CDF = <ABE (Tower and stick are vertically to ground)

Therefore ΔABE ~ ΔCDF (By AAA similarity)

So, AB/CD = BE/DF

AB/6 = 28/4

AB/6 = 7

AB = 7 x 6

AB = 42 m

So, height of tower will be 42 meter.

25. Question

In Fig. 4.145 (a) ΔABC is right angled at C and $DE \perp AB$. Prove that $\Delta ABC \sim \Delta ADE$ and hence find the lengths of AE and DE.

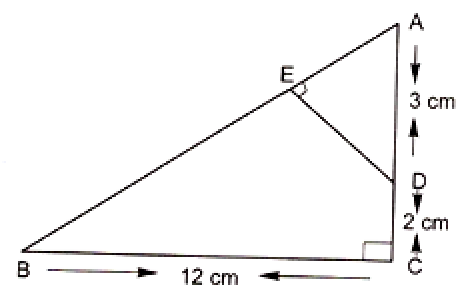


Fig. 4.145 (a)

Answer

In ΔABC, by Pythagoras theorem

$AB^2 = AC^2 + BC^2$

Or, $AB^2 = 5^2 + 12^2$

Or, $AB^2 = 25 + 144$

Or, $AB^2 = = 169$

Or AB = 13 (Square root both side)

In Δ AED and Δ ACB

<A = <A (Common)

<AED = <ACB (Each 90°)

Then, Δ AED ~ Δ ACB(Corresponding parts of similar triangle are proportional)

So, AE/AC = DE/ CB =AD/ AB

Or, AE/5 = DE/12 = 3/13

Or, AE/5 = 3/13 and DE/12 = 3/13

Or, AE = 15/13cm and DE = 36/13cm

Exercise 4.6

1. Question

Triangles ABC and DEF are similar.

(i) If area (ΔABC) = 16 cm^2, area (ΔDEF) = 25 cm^2 and BC = 2.3 cm, find EF.

(ii) If area (ΔABC) = 9 cm^2, area (ΔDEF) = 64 cm^2 and DE = 5.1 cm, find AB.

(iii) If AC = 19 cm and DF = 8 cm, find the ratio of the area of two triangles.

(iv) If area (ΔABC) = 36 cm^2, area (ΔDEF) = 64 cm^2 and DE = 6.2 cm, find AB.

(v) If AB = 1.2 cm and DE = 1.4 cm, find the ratio of the areas of $\Delta ABC \text{ and } \Delta DEF$.

Answer

(i) We have

ΔABC ~ΔDEF

Area (ΔABC) = 16cm^2

Area (ΔDEF) = 25cm^2

And BC = 2.3cm

Since, ΔABC ~ΔDEF

Then, Area (ΔABC)/Area (ΔDEF)

= BC^2/EF^2 (By are of similar triangle theorem)

Or, 16/25 = $(23)^2/ EF^2$

Or, 4/5 = 2.3/EF (By taking square root)

Or, EF = 11.5/4

Or, EF = 2.875cm

(ii) We have

ΔABC ~ΔDEF

Area (ΔABC) = 9cm^2

Area (ΔDEF) = 64cm^2

And BC = 5.1cm

Since, ΔABC ~ΔDEF

Then, Area (ΔABC)/Area (ΔDEF)

= AB^2/DE^2 (By are of similar triangle theorem)

Or, 9/64 = $AB^2/(5.1)^2$

Or, AB = 3 x 5.1/8 (By taking square root)

Or, AB = 1.9125cm

(iii) We have,

ΔABC ~ ΔDEF

AC = 19cm and DF = 8cm

By area of similar triangle theorem

Then, Area of ΔABC/Area of ΔDEF = AC^2 /DE^2(Br area of similar triangle theorem)

$(19)^2/(8)^2$ = 364/64

(iv) We have

Area ΔABC = 36cm^2

Area ΔDEF = 64 cm^2

DE = 6.2 cm

And , ΔABC ~ΔDEF

By area of similar triangle theorem

Area of ΔABC/Area of ΔDEF = AB^2 /DE^2

Or, 36/64 = 6x 6.2/8 (By taking square root)

Or, AB = 4.65cm

(V) We have

ΔABC ~ ΔDEF

AB = 12cm and DF = 1.4 cm

By area of similar triangle theorem

Area of ΔABC/Area of ΔDEF = AB^2 /DE^2

Or, $(1.2)^2/(1.4)^2$ = 1.44x/1.96

2. Question

In Fig. 4.177, $\Delta ACB \sim \Delta APQ$. If BC = 10 cm, PQ = 5 cm, BA = 6.5 cm and AP = 2.8 cm, find CA and AQ. Also, find the area (ΔACB) : area (ΔAPQ).

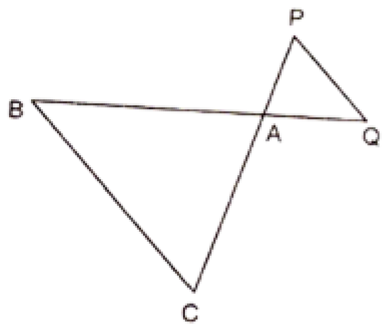


Fig. 4.177

Answer

We have,

ΔACB ~ ΔAPQ

Then, AC/AP = CB/PQ = AB/AQ[Corresponding parts of similar Δ are proportional]

Or, AC/2.8 = 10/5 = 6.5/AQ

Or, AC/2.8 = 10/5 and 10/5 = 6.5/AQ

Or, AC = 5.6cm and AQ = 3.25cm

By area of similar triangle theorem

Area of ΔACB/Area of ΔAPQ = BC^2 /PQ^2

= $(10)^2/(5)^2$

= 100/25

= 4 cm

3. Question

The areas of two similar triangles are 81 cm^2 and 49 cm^2 respectively. Find the ratio of their corresponding heights. What is the ratio of their corresponding medians?

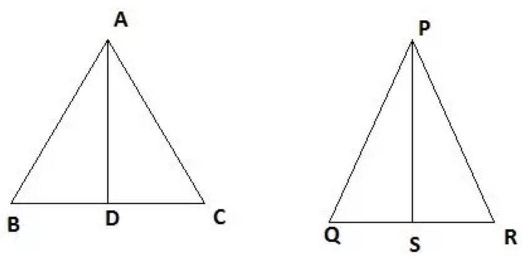
Answer

Given : ΔABC ~ ΔPQR

Area (ΔABC) = 81 cm^2

Area (ΔPQR) = 49 cm^2

Figure:



And AD and PS are the altitudes

By area of similar triangle theorem: The ratio of the areas of two similar triangles equal to the ratio of squares of the corresponding sides of triangles.

$$\frac{Area\ of\ \triangle\ ABC}{Area\ of\ \triangle\ PQR} = \frac{AB^2}{PQ^2}$$

$$\frac{AB^2}{PQ^2} = \frac{81}{49}$$

$$\frac{AB}{PQ} = \sqrt{\frac{81}{49}}$$

$$\frac{AB}{PQ} = \frac{9}{7}$$

We also know that:

$$\frac{AD}{PS} = \frac{AB}{PQ}$$

$$Therefore, \frac{AD}{PS} = \frac{9}{7}$$

So, Ratio of altitudes = 9/7

Hence, ratio of altitudes = Ratio of medians = 9:7

4. Question

The areas of two similar triangles are 169 cm^2 and 121 cm^2 respectively. If the longest side of the larger triangle is 26 cm, find the longest side of the smaller triangle.

Answer

We have,

ΔABC ~ Δ PQR

Area (ΔABC) = 169cm^2

Area (PQR) = 121 cm^2

And AB = 26 cm

By area of similar triangle theorem

Area of ΔABC/Area of ΔPQR = AB^2 /PQ^2

Or, 169/125 = 26^2/ PQ^2

Or, 13/11 = 26/PQ (Taking square root)

Or, PQ = 11/13 x 26

Or, PQ = 22cm

5. Question

Two isosceles triangles have equal vertical angles and their areas are in the ratio 36 : 25.. Find the ratio of their corresponding heights.

Answer

Given : - AB = AC, PQ = PR and <A = <P

And AD and PS are altitudes

And, Area (ΔABC)/Area of(ΔPQR) = 36/25……………….(i)

To find: AD/PS

Proof:- Since, AB = AC and PQ = PR

Then, AB/AC = 1 and PQ/PR = 1

So, AB/AC = PQ/PR

Or, AB/PQ = AC/PR……………….(ii)

In ΔABC and ΔPQR

<A = <P (Given)

AB/PQ = AC/PR (From equation ii)

Then, ΔABC ~ ΔPQR (BY AA similarity)

So, Area of ΔABC/Area of ΔPQR = AB^2 /PQ^2…..(iii) (By area of similar triangle)

Compare equation I and II

AB^2/PQ^2 = 36/25

Or, AB/PQ = 6/5

In ΔABD and ΔPQS

<B = <Q (ΔABC ~ ΔPQR)

<ADB = <PSO (Each 90°)

Then , ΔABD ~ ΔPQS (By AA similarity)

So, AB/ PQ = AD/PS

6/5 = AD/ PS (From iv)

6. Question

The areas of two similar triangles are 25 cm^2 and 36 cm^2 respectively. If the altitude of the first triangle is 2.4 cm, find the corresponding altitude of the other.

Answer

We have,

ΔABC ~ Δ PQR

Area (ΔABC) = 25 cm^2

Area (PQR) = 36 cm^2

And AD = 2.4 cm

And AD and PS are the altitudes

To find: PS

Proof: Since, ΔABC ~ ΔPQR

Then, by area of similar triangle theorem

Area of ΔABC/Area of ΔPQR = AB^2 /PQ^2

25/36 = AB^2/PQ^2

5/6 = AB/PQ………………..(i)

In ΔABD and Δ PQS

<B = <Q (ΔABC ~ ΔPQR)

<ADB = <PSQ (Each 90°)

Then, ΔABD ~ Δ PQS (By AA similarity)

So, AB/PS = AD/PS…………(ii) (Corresponding parts of similar Δ are proportional)

Compare (i) and (ii)

AD/PS = 5/6

2.4/PS = 5/6

PS = 2.4 x 6/5

PS = 2.88cm

7. Question

The corresponding altitudes of two similar triangles are 6 cm and 9 cm respectively. Find the ratio of their areas.

Answer

We have,

ΔABC ~ ΔPQR

AD = 6cm

PS = 9cm

By area of similar triangle theorem

Area of ΔABC/Area of ΔPQR = AB^2 /PQ^2............(i)

In ΔABD and ΔPQS

<B = <Q (ΔABC ~ ΔPQS)

<ADB = <PSQ (Each 90°)

Then, ΔABD ~ ΔPQS (By AA Similarity)

So, AB/PQ = AD/PS (Corresponding parts of similar Δ are proportional)

Or, AB/PQ = 6/9

Or, AB/PQ = 2/3 ....(ii)

Compare equation (i) and (ii)

Area of ΔABC/Area of ΔPQR = $(2/3)^2$ = 4/9

8. Question

ABC is a triangle in which $\angle A$ = 90°, $AN \perp BC$, BC = 12 cm and AC = 5 cm. Find the ratio of the areas of ΔANC and ΔABC.

Answer

In Δ ANC and Δ ABC

<C = <C (Common)

<ANC = <BAC (Each 90°)

Then, Δ ANC ~ Δ ABC (By AA similarity)

By area of similarity triangle theorem.

Area of ΔABC/Area of ΔPQR = AC^2 /BC^2

Or, $5^2/12^2$

Or, 25/144

9. Question

In Fig. 4.178, $DE \parallel BC$

(i) If DE = 4 cm, BC = 6 cm and area (ΔADE) = 16 cm^2, find the area of ΔABC.

(ii) If DE = 4 cm, BC = 8 cm and area (ΔADE) = 25 cm^2, find the area of ΔABC.

(iii) If DE : BC = 3 : 5. Calculate the ratio of the areas of ΔADE and the trapezium BCED.

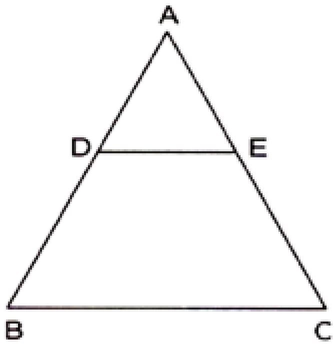


Fig. 4.178

Answer

(i) We have , DE||BC, DE = 4cm, BC = 6cm and area (ΔADE) = $16cm^2$

In ΔADE and ΔABC

<A = <A (Common)

<ADE = <ABC (Corresponding angles)

Then, ΔADE ~ ΔABC (BY AA similarity)

So, By area of similar triangle theorem

Area of ΔADE/Area of ΔABC = DE^2 /BC^2

16/Area of ΔABC = $4^2/6^2$

Or, Area (ΔABC) = 16 x 36/16

= $36cm^2$

(ii) We have , DE||BC, DE = 4cm, BC = 8cm and area (ΔADE) = $25cm^2$

In ΔADE and ΔABC

<A = <A (Common)

<ADE = <ABC (Corresponding angles)

Then, ΔADE ~ ΔABC (BY AA similarity)

So, By area of similar triangle theorem

Area of ΔADE/Area of ΔABC = DE^2 /BC^2

25/Area of ΔABC = $4^2/8^2$

Or, Area (ΔABC) = 25 x 64/16

= 100 cm^2

(iii) We have DE||BC, And DE/BC = 3/5 ....(i)

In ΔADE and ΔABC

<A = <A (Common)

<ADE = <ABC (Corresponding angles)

Then, ΔADE ~ ΔABC (BY AA similarity)

So, By area of similar triangle theorem

Area of ΔADE/Area of ΔABC = DE^2 /BC^2

Area of ΔADE/Area of ΔADE + Area of trap. DECB = $3^2/5^2$

Or, 25 area ΔADE = 9 Area of ΔADE +9 Area of trap. DECB

Or 25 area ΔADE - 9 Area of ΔADE = 9 Area of trap. DECB

Or, 16 area ΔADE = 9 Area of trap. DECB

Or, area ΔADE / Area of trap. DECB = 9/16

10. Question

In $\triangle ABC$, D and E are the mid-points of AB and AC respectively. Find the ratio of the areas of $\triangle ADE \text{ and } \triangle ABC$.

Answer

We have, D and E as the midpoint of AB and AC

So, according to the midpoint therom

DE||BC and DE = 1/2 BC.................(i)

In ΔADE and ΔABC

<A = <A (Common)

<ADE = <B (Corresponding angles)

Then, ΔADE ~ ΔABC (By AA similarity)

By area of similar triangle theorem

Area ΔADE/ Area ΔABC = DE^2/BC^2

Or, $(1/2BC)^2/(BC)^2$

Or, 1/4

11. Question

In Fig. 4.179, $\Delta\ ABC\ and\ \Delta\ DBC$ are on the same base BC. If AD and BC intersect at O. Prove that

$$\frac{Area\ (\Delta\ ABC)}{Area\ (\Delta\ DBC)} = \frac{AO}{DO}$$

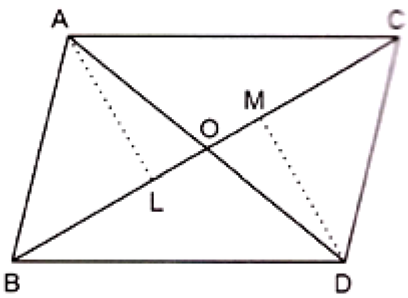


Fig. 4.179

Answer

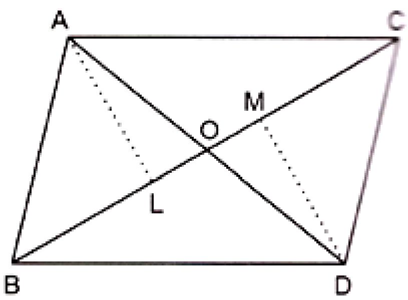


Fig. 4.179

We know that area of a triangle = 1/2 x base x height

Since, ΔABC and ΔDBC are one same base.

Therefore ratio between their areas will be as ratio of their heights.

Let us draw two perpendiculars AP and DM on line BC

In ΔALO and ΔDMO,

∠ALO =∠DMO (Each is 90°)

∠AOL = ∠DOM (Vertically opposite angle)

∠OAL = ∠ODM (remaining angle)

Therefore ΔALO ~ ΔDMO (By AAA rule)

Therefore AL/DM = AO/DO

Therefore, $\frac{Area\ (\Delta\ ABC)}{Area\ (\Delta\ DBC)} = \frac{AO}{DO}$

12. Question

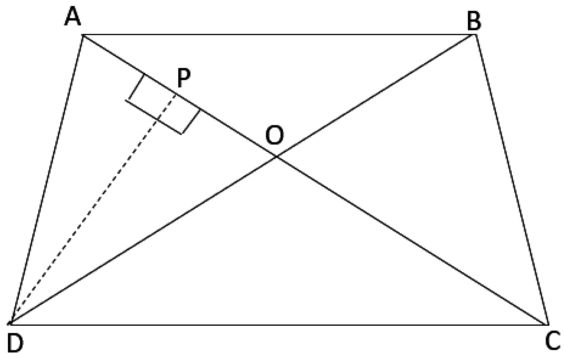
ABCD is a trapezium in which $AB \parallel CD$. The diagonals AC and BD intersect at O. Prove that : (i) $\Delta\ AOB \sim \Delta\ COD$

(ii) If OA = 6 cm, OC = 8 cm, Find:

(a) $\frac{Area\ (\Delta\ AOB)}{Area\ (\Delta\ COD)}$ (b) $\frac{Area\ (\Delta\ AOD)}{Area\ (\Delta\ COD)}$

Answer

We have,



AB||DC

In ΔAOB and ΔCOD ∠AOB =∠COD (Vertically opposite angles)

∠OAB = ∠OCD (Alternate interior angle)

Then , ΔAOB ~ ΔCOD (By AA similarity)

(a) By area of similar triangle theorem.

$$\frac{Area\ of\ \Delta AOB}{Area\ of\ \Delta COD} = \frac{OA^2}{OC^2}$$

$$\Rightarrow \frac{Area\ of\ \Delta AOB}{Area\ of\ \Delta COD} = \frac{6^2}{8^2}$$

$$\Rightarrow \frac{Area\ of\ \Delta AOB}{Area\ of\ \Delta COD} = \frac{36}{64} = \frac{9}{16}$$

b) Draw DP ⊥ AC

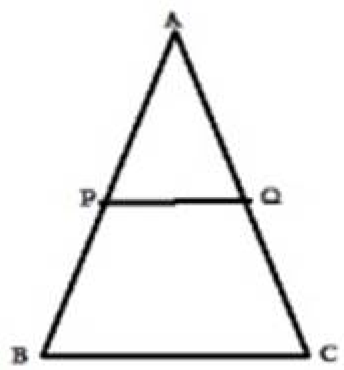
$$\Rightarrow \frac{Area\ of\ \Delta AOD}{Area\ of\ \Delta COD} = \frac{\frac{1}{2}\times OA\times DP}{\frac{1}{2}\times OC\times DP}$$

$$\Rightarrow \frac{Area\ of\ \Delta AOD}{Area\ of\ \Delta COD} = \frac{6}{8} = \frac{3}{4}$$

13. Question

In ΔABC, P divides the side AB such that AP : PB = 1 : 2. Q is a point in AC such that $PQ \parallel BC$. Find the ratio of the areas of ΔAPQ and trapezium BPQC.

Answer



We know

PQ∥BC

1= AP

2 PB

In $\Delta APQ\ and\ \Delta ABC$

∠A=∠A [Common]

∠APQ=∠B [Corresponding angle]

ΔABC~ ΔAPQ

Area(ΔAPQ) =AP^2

Area (ΔABC) AB^2

ar (ΔAPQ)______________ = $1^{2}/3^{2}$

ar(ΔAPQ)+ar($trapBPQC$)

9ar(ΔAPQ)= ar(ΔAPQ)+ar($trapBPQC$)

9ar(ΔAPQ)- ar(ΔAPQ)=ar($trapBPQC$)

8ar(ΔAPQ)=ar($trapBPQC$)

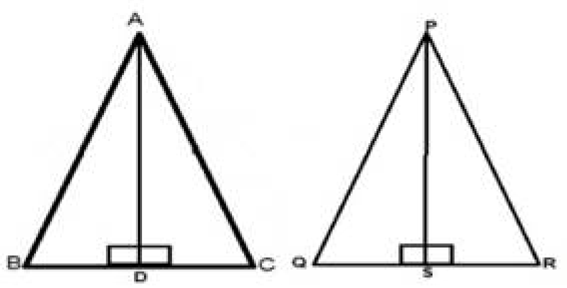
ar(ΔAPQ) = $\frac{1}{8}$

ar($trapBPQC$)

14. Question

The areas of two similar triangles are 100 cm^2 and 49 cm^2 respectively. If the altitude of the bigger triangle is 5 cm, find the corresponding altitude of the other.

Answer



We have,

ΔABC~ ΔPQR

Area (ΔABC) =100cm^2

Area (ΔPQR) =49 cm^2

AD= 5cm

AD and PS are the altitudes

by area of similar triangle theorem

Area(ΔABC) =AB^2

Area (ΔPQR) PQ^2

AB^2 = 100/49

PQ^2

AB/PQ= 10/7 ....(i)

In ΔABD and ΔPQS

∠B=∠Q [ΔABC~ ΔPQR]

∠ADB=∠PQS=90°

ΔABD ~ ΔPQS [By AA similarity]

AB/PQ=AD/PS ....(ii)

Compare equ. (i)and(ii)

AD/PS=10/7

5/PS=10/7

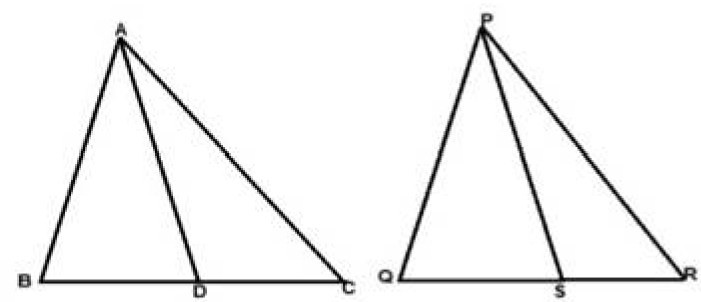
PS=35/10

PS=3.5 cm

15. Question

The areas of two similar triangles are 121 cm^2 and 64 cm^2 respectively. If the median of the first triangle is 12.1 cm, find the corresponding median of the other.

Answer



We have,

ΔABC~ ΔPQR

Area (ΔABC) =121cm^2

Area (ΔPQR) =64cm^2

AD= 12.1cm

AD and PS are the medians

By area of similar triangle theorem

Area(ΔABC) =AB^2

Area (ΔPQR) PQ^2

AB^2 =121

PQ^2 64

AB =11 .... (i)

PQ 8

ΔABC~ ΔPQR

AB/PQ=BC/QR [Corresponding parts of similar triangles are proportional] AB/PQ=2BD/2QS [AD and BD are medians]

AB/PQ=BD/QS ………… (ii)

In ΔABD and ΔPQS

∠B=∠Q [ΔABC$\sim$ ΔPQS]

AB/PQ=BD/QS [from (ii)]

ΔABD ~ ΔPQS [By AA similarity]

AB/PQ=AD/PS Compare equ. (i)and(ii)

AD/PS=11/8

12.1/PS=11/8

PS=12.1x8/8

PS= 8.8 cm

16. Question

If $\Delta ABC \sim \Delta DEF$ such that AB = 5 cm, area (ΔABC) = 20 cm^2 and area (ΔDEF) = 45 cm^2 , determine DE.

Answer

We have

ΔABC$\sim$ ΔDEF

Where AB= 5cm

Area (ΔABC) =20cm^2

Area (ΔDEF) =45cm^2

By area of similar triangle theorem

Area (ΔABC) =AB^2

Area (ΔDEF) DE^2

5^2/DE^2=20/25

25/DE^2=4/9

5/DE=2/3

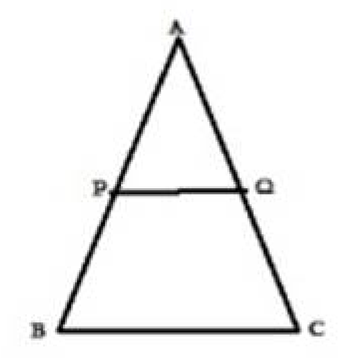
DE=3x5/2

DE=7.5 cm

17. Question

In $\triangle ABC$, PQ is a line segment intersecting AB at P and AC at Q such that $PQ \parallel BC$ and PQ divides $\triangle ABC$ into two parts equal in area. Find $\frac{BP}{AB}$.

Answer



We know

PQ∥BC

Area (ΔAPQ) =Area ($trapPQCB$)

Area (ΔAPQ) =Area (ΔABC)- Area (ΔAPQ)

2Area (ΔAPQ)= Area (ΔABC) ....(i)

In $\Delta APQ \ and \ \Delta ABC$

∠A=∠A [Common]

∠APQ=∠B [Corresponding angle]

ΔABC~ ΔAPQ

Area(ΔAPQ) =AP^2

Area (ΔABC) AB^2

Area(ΔAPQ) =AP^2

Area (ΔAPQ) AB^2 [By using (I)]

1= AP^2

2 AB^2

$\frac{1}{\sqrt{2}}$ =AP/AB

$\frac{1}{\sqrt{2}} = \frac{AB-BP}{\text{AB}}$

$\frac{1}{\sqrt{2}} = \frac{AB}{AB} - \frac{BP}{AB}$

$\frac{1}{\sqrt{2}}$=1-BP/AB

BP/AB=1-$\frac{1}{\sqrt{2}}$

BP/AB= $\frac{\sqrt{2}-1}{\sqrt{2}}$

18. Question

The areas of two similar triangles ABC and PQR are in the ratio 9 : 16. If BC = 4.5 cm, find the length of QR.

Answer

We have,

ΔABC$\sim$ ΔPQR

Area(ΔABC) =BC^2

Area (ΔPQR) QR^2

$(4.5)^2/QR^2$=9/16

4.5/QR=3/4

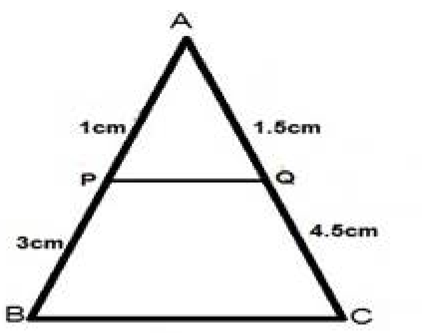
QR=4x4.5/3

QR=6cm

19. Question

ABC is a triangle and PQ is a straight line meeting AB in P and AC in Q. If AP = 1 cm, PB = 3 cm, AQ = 1.5 cm, QC = 4.5 m, prove that area of ΔAPQ is one-sixteenth of the area of ΔABC.

Answer



AP=1 cm, PB=3 cm,AQ=1.5cm,and QC=4.5 m

In ΔAPQ and ΔABC

$\angle A=\angle A$ [Common]

AP/AB=AQ/AC [Each equal to 1/4]

ΔAPQ~ ΔABC [By SAS]

By area of similar triangle theorem

$$\frac{Area\ of\ \triangle APQ}{Area\ of\ \triangle ABC} = (\frac{AP}{AB})^2$$

$$\frac{Area\ of\ \triangle APQ}{Area\ of\ \triangle ABC} = (\frac{1}{4})^2$$

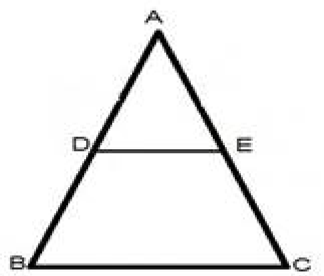
$$\frac{Area\ of\ \triangle APQ}{Area\ of\ \triangle ABC} = \frac{1}{16}$$

Area (ΔABC) = 16 x ar(ΔABC)

20. Question

If D is a point on the side AB of ΔABC such that AD : DB = 3.2 and E is a point on BC such that $DE \parallel AC$. Find the ratio of areas of ΔABC *and* ΔBDE.

Answer



We have

AD/DB=3/2

In ΔBDE and ΔBAC

∠B=∠B [Common]

∠BDE=∠A [Corresponding]

ΔBDE~ ΔBAC

Area(ΔABC) =AB^2

Area (ΔBDE) BD^2

=$5^2/2^2$ [AD/DB=3/2]

=25/4

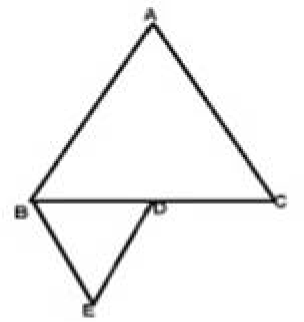
Area(ΔABC)

Area (ΔBDE) =25:4

21. Question

If $\Delta\ ABC\ and\ \Delta\ BDE$ are equilateral triangles, where D is the mid point of BC, find the ratio of areas of $\Delta\ ABC\ \ and\ \Delta\ BDE$.

Answer



ΔABC and ΔBDE is an equilateral triangles

ΔABC$\sim$ ΔDEF [By SAS]

By area of similar triangle theorem

Area(ΔABC) =AB^2 [D is the midpoint of BC]

Area (ΔBDE) BD^2

=4BD^2/BD^2

=4/1

Area(ΔABC) = 4:1 Area (ΔBDE)

22. Question

AD is an altitude of an equilateral triangle ABC. On AD as base, another equilateral triangle ADE is constructed. Prove that Area ($\Delta\ ADE$) : Area ($\Delta\ ABC$) = 3 : 4.

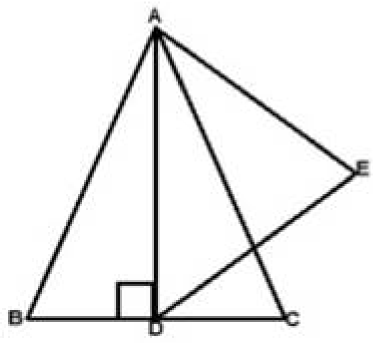
Answer

Given: AD is an altitude of an equilateral triangle ABC. On AD as base, another equilateral triangle ADE is constructed

To prove: Area ($\Delta\ ADE$) : Area ($\Delta\ ABC$) = 3 : 4.

Proof:

Construct the figure according to the conditions given.



We have,

ΔABC is an equilateral triangle

Let one side AB be 2XSince in equilateral triangle all the sides are of equal length.

⇒ AB=BC=AC= 2X

∵ AD⊥BCSince perpendicular bisects the given side into two equal parts,then BD=DC=x

Now, In ΔADB

By Pythagoras theorem,$AB^2 = AD^2 + BD^2$

$AD^2=AB^2 - BD^2 AD^2 = (2x)^2-(x)^2 AD^2 = 3x^2$

AD=$\sqrt{3x}$ cm

ΔABC and ΔADE both are equilateral triangles

Since, all the angles of the equilateral triangle are of 60°.

∴ ΔABC∼ ΔADE [By AA similarity]

By the theorem which states that the areas of two similar triangles are in the ratioof the squares of the any two corresponding sides.

$$\frac{Area(\triangle ADE)}{Area(\triangle ABC)} = \frac{AD^2}{AB^2}$$

$$\frac{Area(\triangle ADE)}{Area(\triangle ABC)} = \frac{(\sqrt{3x})^2}{(2x)^2}$$

$$\frac{Area(\triangle ADE)}{Area(\triangle ABC)} = \frac{3x^2}{4x^2}$$

$$\frac{Area(\triangle ADE)}{Area(\triangle ABC)} = \frac{3}{4}$$

Hence,Area ($\triangle ADE$) : Area ($\triangle ABC$) = 3 : 4

Exercise 4.7

1. Question

If the sides of a triangle are 3 cm, 4 cm and 6 cm long, determine whether the triangle is a right-angled triangle.

Answer

We have,

AB=3cm, BC=4cm, AC=6cm

∴ $AB^2 = 3^2 = 9$

$BC^2 = 4^2 = 16$

$AC^2 = 6^2 = 36$

Since $AB^2 + BC^2 \neq AC^2$

SO Triangle is not a right angle.

2. Question

The sides of certain triangles are given below. Determine which of them are right triangles.

(i) a = 7 cm, b = 24 cm and c = 25 cm

(ii) a = 9 cm, b = 16 cm and c = 18 cm

(iii) a = 1.6 cm, b = 3.8 cm and c = 4 cm

(iv) a = 8 cm, b = 10 cm and c = 6 cm

Answer

(i) a= 7, b= 24, c=25

Here $a^2=49$, $b^2=576$, $c^2=625$

$=a^2+b^2$

=49+576

$=625=c^2$

∴ So given triangle is a right angle.

(ii) a=9, b=16, c= 18

Here $a^2=81$, $b^2= 256$, $c^2= 324$

$=a^2+b^2$

$=81+256$

$=337 \neq c^2$

So given Triangle is not a right angle.

(iii) $a=1.6$, $b=3.8$, $c= 4$

Here $a^2=2.56$, $b^2= 14.44$, $c^2= 16$

$=a^2+b^2$

$=2.56+14.44$

$=17 \neq c^2$

So given Triangle is not a right angle.

(iv) $a=8$, $b=10$, $c= 6$

Here $a^2=64$, $b^2= 100$, $c^2= 36$

$=a^2+c^2$

$=64+36$

$=100 = b^2$

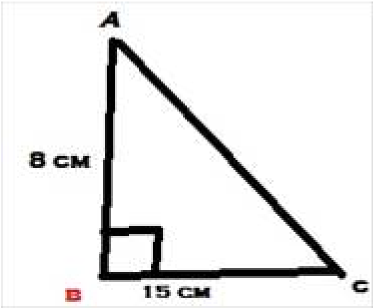
So given Triangle is a right angle.

3. Question

A man goes 15 metres due west and then 8 metres due north. How far is he from the starting point?

Answer

Let the man starts walk from point A and finished at



Point C.

∴ In ⊿ ABC

SO $AC^2=AB^2+BC^2$

$AC^2=8^2+15^2$

$AC^2=64+225$

$AC^2=289$

$AC=\sqrt{289}$

$AC=17$ m

The man is 17 m far from the starting point.

4. Question

A ladder 17 m long reaches a window of a building 15 m above the ground. Find the distance of the foot of the ladder from the building.

Answer

In ⊿ ABC

$AC^2=AB^2+BC^2$

$17^2=15^2+BC^2$

$289=225+BC^2$

$BC^2=289-225$

$BC^2=64$

$BC=\sqrt{64}$

$BC=8$ m

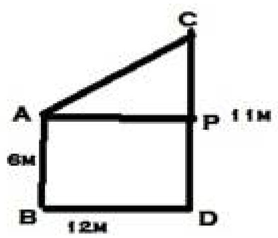
Distance of the foot of ladder is 8 m from the building.

5. Question

Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.

Answer

Let AB and CD be the poles.



AB=PD= 6m, CD=11m

BD=AP=12m

CP=CD-PD

CP=11-6

CP=5

In ⊿ APC

$AC^2=CP^2+AP^2$

$AC^2=12^2+5^2$

$AC^2=144+25$

$AC^2=169$

AC=$\sqrt{169}$

AC= 13m

6. Question

In an isosceles triangle ABC, AB = AC = 25 cm, BC = 14 cm. Calculate the altitude from A on BC.

Answer

We have,

AB=AC=25cm

BC=14cm

In ⊿ ACD and ⊿ ABD

∠ADB=∠ADB=90

AB=AC=25cm

AD=AD (Common)

⊿ ABD≅∠ACD

∴BD=CD=7cm (By c.p.c.t)

In ⊿ACD

$AB^2=AD^2+BD^2$

$25^2=AD^2+72$

$625=AD^2+49$

$AD^2=625-49$

$AD^2=576$

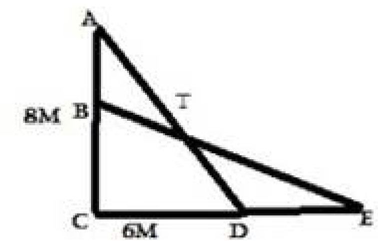
AD=$\sqrt{576}$

AD=24 cm

7. Question

The foot of a ladder is 6 m away from a wall and its top reaches a window 8 m above the ground. If the ladder is shifted in such a way that its foot is 8 m away from the wall, to what height does its tip reach?

Answer



Let length of ladder be AD=BE=1m

In Δ ACD

$AD^2=AC^2+CD^2$

$t^2= 8^2+6^2$ ………………..(i)

In Δ BCE

$BE^2=BC^2+EC^2$

$t^2= BC^2+8^2$ …………….. (II)

From (i) and (ii)

$BC^2+8^2=8^2+6^2$

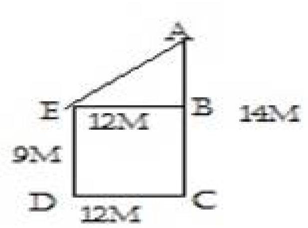
$BC^2=6^2$

BC=6m

8. Question

Two poles of height 9 m and 14 m stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.

Answer



We have,

AC=14m, DC=12m, ED=BC=9m

Draw EB ⊥ AC

∴ AB=AC-BC

AB= 14-9=5m

EB=DC=12m

In Δ ABE

$AE^2=AB^2+BE^2$

$AE^2=5^2+12^2$

$AE^2=25+144$

$AE^2=169$

AE=$\sqrt{169}$

AE=13m

9. Question

Using Pythagoras theorem determine the length of AD in terms of b and c shown in Fig 4.219.

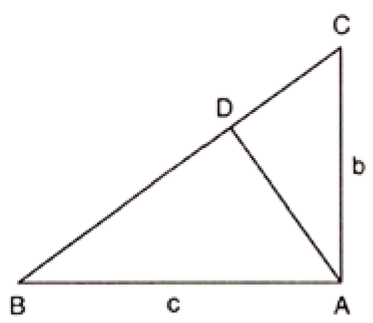


Fig. 4.219

Answer

In Δ ABC

$BC^2=AB^2+AC^2$

$BC^2=c^2+b^2$

BC= $\sqrt{c2+b2}$ ....(i)

In Δ ABC and In Δ CBA

∠B= ∠B (Common)

∠ADB=∠BAC=90°

∴Δ ABD ~ Δ CBA

∴ AB/CB=AD/CA

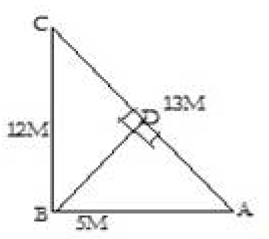
$c/\sqrt{c2+b2}$ =AD/b

AD=bc/$\sqrt{c2+b2}$

10. Question

A triangle has sides 5 cm, 12 cm and 13 cm. Find the length to one decimal place, of the perpendicular from the opposite vertex to the side whose length is 13 cm.

Answer



Here AB=5cm,BC=12cm, AC=13cm.

AC2=AB^2+BC^2

Δ ABC is a right angled triangle at ∠B.

Area Δ ABC=1/2(BCxBA)

=1/2(12x5)

=1/2x60

=$30cm^2$

Also Area of Δ ABC=1/2xACxBD

=1/2(13xBD)

30=1/2(13xBD)

13XBD=30x2

BD=60/13

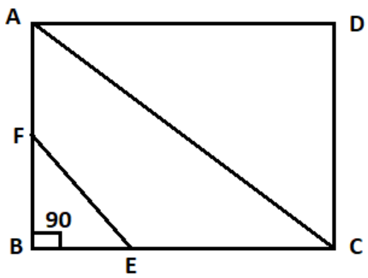
BD=4.6 cm

11. Question

ABCD is a square. F is the mid-point of AB. BE is one third of BC. If the area of ΔFBE = 108 cm^2, find the length of AC.

Answer

According to the question, the figure is :



∵ ABCD is a square. Hence, AB = BC = CD = DA

∵ F is the midpoint of AB.

∴ Length of BF = AB/2 = BC/2 (∵ AB = BC)

Given that, BE = BC/3

In ΔFBE, ∠B = 90° and Area of ΔFBE = 108 cm^2

$\therefore \frac{1}{2} \times BE \times BF = 108$

$\Rightarrow \frac{1}{2} \times \frac{BC}{3} \times \frac{BC}{2} = 108$

⇒ BC^2 = 108 × 12

⇒ BC^2 = 36 × 36

⇒ BC = 36 cm^2

AC is the diagonal of the ABCD.

$\therefore$ Length of AC $= \sqrt{BC^2 + AB^2}$

$\Rightarrow AC = \sqrt{36^2 + 36^2}$

$\Rightarrow AC = \sqrt{36^2 + 36^2}$

⇒ AC = 36√2 = 50.904 cm

12. Question

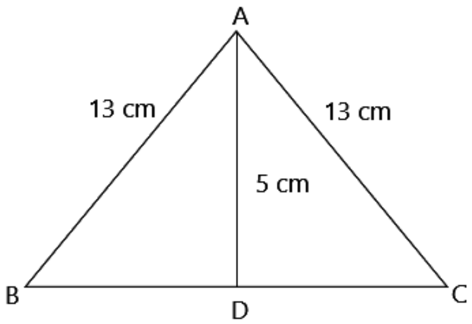
In an isosceles triangle ABC, if AB = AC = 13 cm and the altitude from A on BC is 5 cm, find BC.

Answer

Given: isosceles triangle ABC, where AB = AC = 13 cm and the altitude from A on BC is 5 cm.

To find: The value of BC.

Solution:



In ΔADB

$AD^2+BD^2=AB^2$

$5^2+BD^2=13^2$

$25+BD^2=169$

$BD^2=169-25$

$BD^2=144$

$BD=\sqrt{144}$

BD=12cm

In ΔADB and ΔADC

$\angle ADB=\angle ADC=90^\circ$

AB=AC=13cm

AD=AD (Common)

$\Delta ADB \cong \Delta ADC$ (By RHS condition)

BD=CD=12cm (c.p.c.t)

As BC=BD+DC

BC=12+12

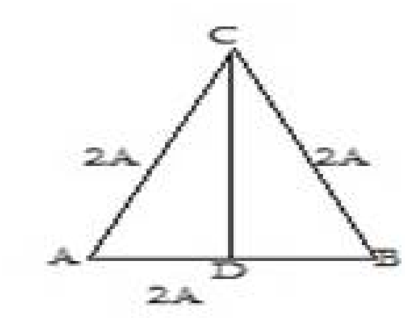
BC = 24cm

13. Question

In a ΔABC, AB = BC = CA = 2 a and $AD \perp BC$. Prove that

(i) $AD = a\sqrt{3}$ (ii) Area (ΔABC) = $\sqrt{3}a^2$

Answer



(i) In Δ ABD and Δ ACD

$\angle ADB = \angle ADC = 90^\circ$

AB=AC (given)

AD=AD (common)

$\Delta ADB \cong \Delta ACD$

BD=CD=a (By c.p.c.t)

In ΔADB

$AD^2+BD^2=AB^2$

$AD^2+a^2=(2a)^2$

$AD^2=4a^2-a^2$

$AD^2=3a^2$

$AD=a\sqrt{3}$

(ii) Area of $\Delta ABC = 1/2 \times BC \times AD$

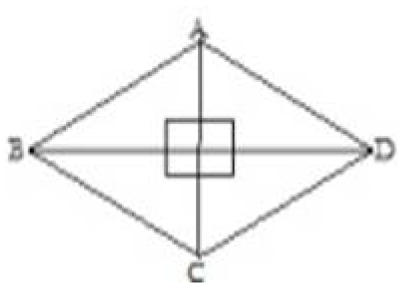
$= 1/2 \times 2a \times a\sqrt{3}$

$=\sqrt{3}a^2$

14. Question

The lengths of the diagonals of a rhombus are 24 cm and 10 cm. Find each side of the rhombus.

Answer



We have,

ABCD is a rhombus

AC and BD are the diagonals with length 10cm and 24 cm respectively.

We know that rhombus of diagonal bisects each other at 90°

∴ AO=OC=5cm and BO=OD=12cm

In ΔAOB

$AB^2=AO^2+BO^2$

$AB^2=5^2+12^2$

$AB^2=25+144$

$AB^2=169$

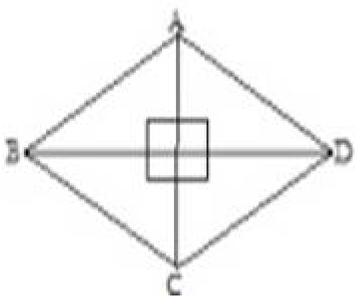
$AB=\sqrt{169}$

AB=13 cm

15. Question

Each side of a rhombus is 10 cm. If one of its diagonals is 16 cm find the length of the other diagonal.

Answer



We have,

ABCD is a rhombus with side 10 cm and diagonal BD=16 CM

We know that rhombus of diagonal bisects each other at 90°

BO=OD=8cm

In ΔAOB

$AB^2=AO^2+BO^2$

$10^2= AO^2 +8^2$

$100= AO^2+64$

$AO^2=100-64$

$AO^2=36$

$AO=\sqrt{36}$

AO=6 cm

∴ AC=AO+OC

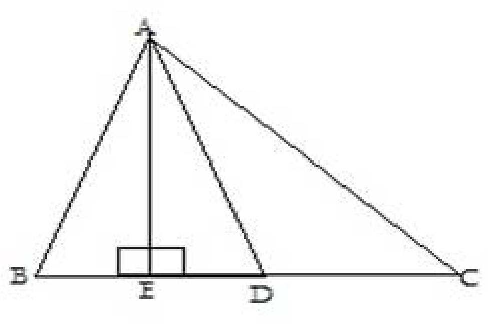
AC=6+6

AC=12 cm

16. Question

In an acute-angled triangle, express a median in terms of its sides.

Answer



We have

In ΔABC, AD is median

AE⊥BC

In ΔAEB

$AB^2=AE^2+BE^2$

$AB^2=AD^2-DE^2+(BD-DE)^2$

$AB^2=AD^2-DE^2+BD^2-2xBDxDE+DE^2$

$AB^2=AD^2+BD^2-2xBDxDE$

$AB^2=AD^2+BC^2/4-BCxDE$ …………. (I) [GIVEN BC=2BD]

In ΔAEC

$AC^2=AE^2+EC^2$

$AC^2=AD^2-DE^2+ (DE+CD)^2$

$AC^2=AD^2-DE^2+2CDxDE$

$AC^2=AD^2+BC^2/4+BCxDE$ ……….(II) [BC=2CD]

By adding equ. (i) and (ii) we get

$AB^2+AC^2=2AD^2+BC^2/2$

$2AB^2+2AC^2=4AD^2+BC^2$ [MULTIPLY BY 2]

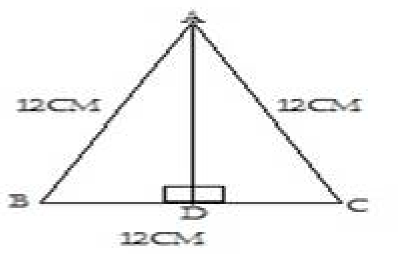
$4AD^2=2AB^2+2AC^2-BC^2$

$AD^2=2AB^2+2AC^2-BC^2$

17. Question

Calculate the height of an equilateral triangle each of whose sides measures 12 cm.

Answer



ΔABC is an equilateral triangle with side 12cm

$AE \perp BC$

In ΔABD and ΔACD

$\angle ADB = \angle ADC = 90°$

AB=AC=12cm

AD=AD (COMMON)

$\Delta ABD \cong \Delta ACD$

$AD^2+BD^2=AB^2$

$AD^2+6^2=12^2$

$AD^2+36=144$

$AD^2=144-36$

$AD^2=108$

$AD=\sqrt{108}$

AD=10.39 cm

18. Question

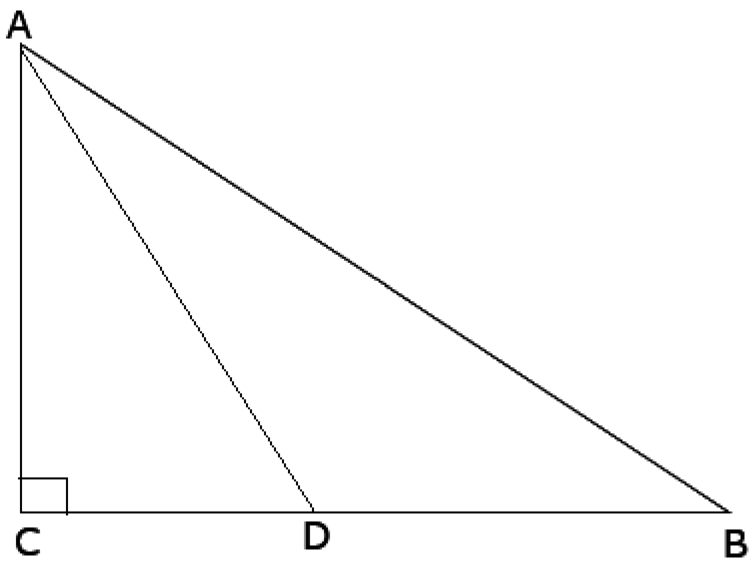
In right-angled triangle ABC in which $\angle C = 90°$, if D is the mid-point of BC, prove that $AB^2 = 4AD^2 - 3AC^2$.

Answer

Given: In right-angled triangle ABC in which $\angle C = 90°$, if D is the mid-point of BC.

To prove: $AB^2 = 4AD^2 - 3AC^2$

Solution:



We have

$\angle C=90°$ and D is the midpoint of BC

In ΔABC

$AB^2=AC^2+BC^2$

As BC = CD + BD D is the mid point of BC$\Rightarrow$ CD = BDSo,$AB^2=AC^2+(CD + CD)^2$

$\Rightarrow AB^2=AC^2+(2CD)^2$

$\Rightarrow AB^2=AC^2+4CD^2$

Also In ΔACD$AD^2=AC^2+CD^2 \Rightarrow CD^2 = AD^2 - AC^2$So,

$\Rightarrow AB^2=AC^2+4(AD^2-AC^2)$

$AB^2=AC^2+4AD^2-4AC^2$

$AB^2=4AD^2-3AC^2$

19. Question

In Fig. 4.220, D is the mid-point of side BC and $AE \perp BC$. If BC = a, AC = b, AB = c, ED = x, AD = p and AE = h, prove that:

(i) $b^2 = p^2 + a + \frac{a^2}{4}$ (ii) $c^2 = p^2 - ax + \frac{a^2}{4}$

(iii) $b^2 + c^2 = 2p^2 + \frac{a^2}{2}$

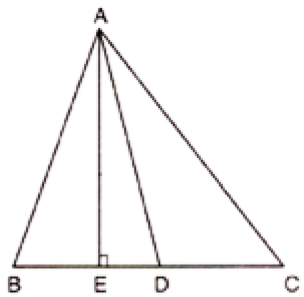


Fig. 4.220

Answer

We have

D is the midpoint of BC

(i) In ΔAEC

$AC^2=AE^2+EC^2$

$b^2=AE^2+(ED+DC)^2$

$b^2=AD^2+DC^2+2xEDxDC$ (Given BC=2CD)

$b^2=p^2+(a/2)^2+2(a/2)x$

$b^2=p^2+a^2/4+ax$

$b^2=p^2 +ax+a^2/4$ .... (i)

(ii) In ΔAEB

$AB^2=AE^2+BE^2$

$c^2=AD^2-ED^2+(BD-ED)^2$

$c^2=p^2-ED^2+BD^2+ED^2-2BDxED$

$c^2=P^2+(a/2)^2-2(a/2)^2x$

$c^2=p^2-ax+a^2/4$ ....(ii)

(iii) Adding equ. (i)and(ii) we get

$b^2+c^2=2p^2+a^2/2$

20. Question

In Fig. 4.221, $\angle B < 90^\circ$ and segment $AD \perp BC$, show that

(i) $b^2 = h^2 + a^2 + x^2 - 2ax$

(ii) $b^2 = a^2 + c^2 - 2ax$

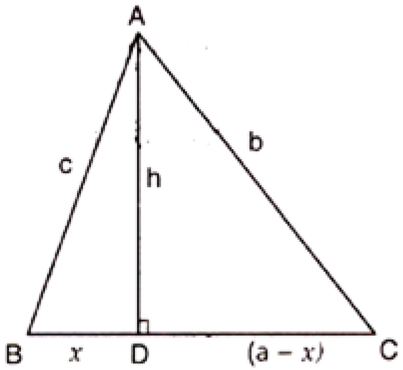


Fig. 4.221

Answer

In ΔADC

$AC^2=AD^2+DC^2$

$b^2=h^2+(a-x)^2$

$b^2=h^2+a^2-2ax+x^2$ .... (i)

$b^2=h^2+x^2-2ax$

$b^2=a^2+ (h^2+x2)-2ax$ (from equ.i)

$b^2=a^2+c^2-2ax$ $[h^2+x^2=c^2]$

21. Question

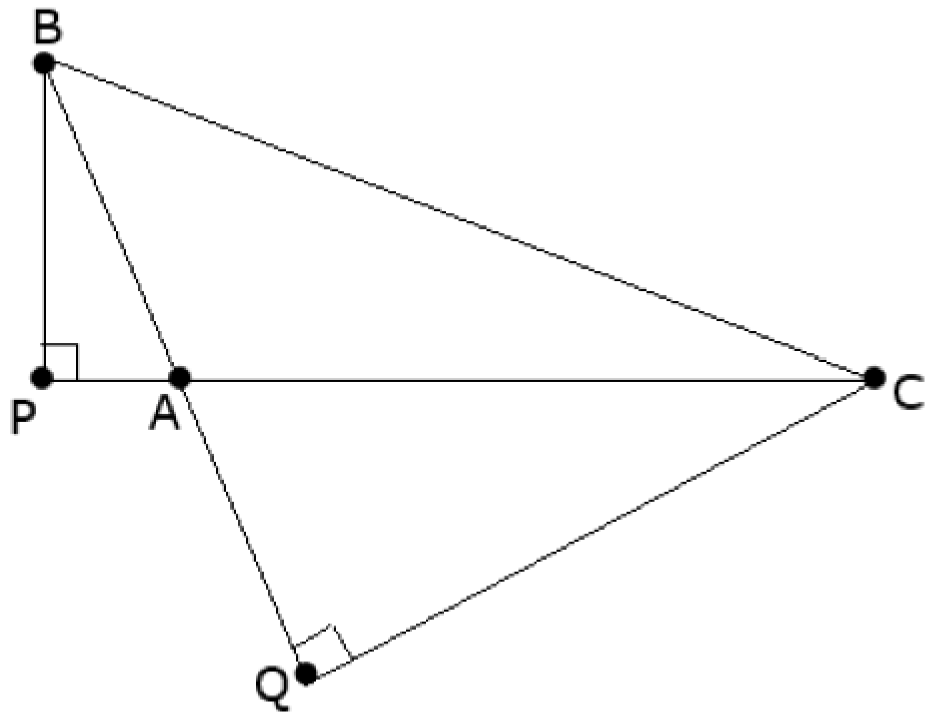
In ΔABC, $\angle A$ is obtuse, $PB \perp AC$ and $QC \perp AB$. Prove that:

(i) $AB \times AQ = AC \times AP$

(ii) $BC^2 = (AC \times CP + AB \times BQ)$

Answer

Draw the diagram according to given questions.



(I) In ⊿APB and ⊿AQC∠A = ∠A

(common)∠P = ∠Q (both 90°)∴ ⊿APB~⊿AQC [By AA similarity]

$\Rightarrow \frac{AP}{AQ} = \frac{AB}{AC}$ {Corresponding part of similar triangle are proportional}

AP x AC = AQ x AB ....(1)

(II)

In ⊿BPCBy pythagoras theoram,

$BC^2 = BP^2 + PC^2$Also in ⊿BPA

$BP^2 = AB^2 - AP^2$Also PC = PA + AC

$\Rightarrow BC^2 = AB^2 - AP^2 + (AP + AC)^2$

Apply the theorem $(a + b)^2 = a^2 + b^2 + 2ab$ in $(AP + AC)^2$

$\Rightarrow BC^2 = AB^2 - AP^2 + AP^2 + AC^2 + 2AP \times AC$

$BC^2 = AB^2 + AC^2 + 2AP \times AC$ ....(ii)

In ⊿BQC

$BC^2 = CQ^2 + BQ^2$

$BC^2 = AC^2 - AQ^2 + (AB + AQ)^2$

$BC^2 = AC^2 - AQ^2 + AB^2 + 2AB \times AQ$

$BC^2 = AC^2 + AB^2 + AQ^2 + 2AB \times AQ$ ....(iii)

Adding equ. (ii)and(iii)

$BC^2 + BC^2 = AB^2 + AC^2 + 2AP \times AC + AC^2 + AB^2 + AQ^2 + 2AB \times AQ$

$\Rightarrow 2BC^2 = 2AC^2 + 2AB^2 + 2AP \times AC + 2AB \times AQ$

$\Rightarrow 2BC^2 = 2AC[AC + AP] + AB[AB + AQ]$

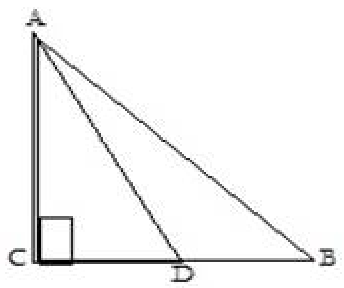
$\Rightarrow 2BC^2 = 2AC \times PC + 2AB \times BQ$

$\Rightarrow BC^2 = AC \times PC + AB \times BQ$Hence proved.

22. Question

In a right ΔABC right-angled at C, if D is the mid-point of BC, prove that $BC^2 = 4(AD^2 - AC^2)$.

Answer



We have

$\angle C=90^\circ$ and D is the midpoint of BC

$LHS=BC^2$

$=(2CD)^2$

$=4CD^2$

$=4(AD^2-AC^2)$ =RHS

23. Question

In a quadrilateral ABCD, $\angle B < 90^\circ$, $AD^2 = AB^2 + BC^2 + CD^2$, prove that $\angle ACD = 90^\circ$.

Answer

We have

$\angle B = 90^\circ$ and

$AD^2 = AB^2 + BC^2 + CD^2$ (Given)

But $AB^2 + BC^2 = AC^2$

$AD^2 = AC^2 + CD^2$

By converse of by Pythagoras

$\angle ACD = 90^\circ$

24. Question

In an equilateral ΔABC, $AD \perp BC$, prove that $AD^2 = 3BD^2$.

Answer

We have $\triangle$ ABC is an equilateral triangle and AD⊥BC

In $\triangle$ ADB$\triangle$ ADC

$\angle ADB = \angle ADC = 90°$ AB=AC (Given)

AD=AD (Common)

$\triangle$ ADB≅$\triangle$ ADC (By RHS condition)

∴ BD=CD=BC/2 .... (i)

In $\triangle$ ABD

$BC^2 = AD^2 + BD^2$

$BC^2 = AD^2 + BD^2$ [Given AB=BC]

$(2BD)^2 = AD^2 + BD^2$ [From (i)]

$4BD^2 - BD^2 = AD^2$

$AD^2 = 3BD^2$

25. Question

ΔABC is a right triangle right-angled at A and $AC \perp BD$. Show that

(i) $AB^2 = BC.BD$ (ii) $AC^2 = BC.DC$

(iii) $AD^2 = BD \cdot CD$ (iv) $\frac{AB^2}{AC^2} = \frac{BD}{DC}$

Answer

(i) In $\triangle$ABD and In $\triangle$CAB

$\angle DAB = \angle ACB = 90°$

$\angle ABD = \angle CBA$ [Common]

$\angle ADB = \angle CAB$ [remaining angle]

So, $\triangle$ADB≅$\triangle$CAB [By AAA Similarity]

∴ AB/CB=BD/AB

$AB^2 = BC \times BD$

(ii)

Let <CAB= x

InΔCBA=180-90°-x

<CBA=90°-x

Similarly in ΔCAD

<CAD=90°-<CAD=90°-x

<CDA=90°-<CAB

=90°-x

<CDA=180°-90°-(90°-x)

<CDA=x

Now in ΔCBA and ΔCAD we may observe that

<CBA=<CAD

<CAB=<CDA

<ACB=<DCA=90°

Therefore ΔCBA~ΔCAD (by AAA rule)

Therefore AC/DC=BC/AC

AC^2=DCxBC

(iii) In DCA and ΔDAB

<DCA=<DAB (both angles are equal to 90°)

<CDA=. <ADB (common)

<DAC=<DBA

ΔDCA= ΔDAB (AAA condition)

Therefore DC/DA=DA/DB

AD^2=BDxCD

(iv) From part (I) AB^2=CBxBD

From part (II) AC^2=DCxBC

Hence AB^2/AC^2=CBxBD/DCxBC

AB^2/AC^2=BD/DC

Hence proved

26. Question

A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Answer

Let OB be the pole and AB be the wire.

$AB^2 = OB^2 + OA^2$

$24^2 = 18^2 + OA^2$

$OA^2 = 576 - 324$

$OA^2 = 252$

$AO = \sqrt{252}$

$AO = 6\sqrt{7}$ m.

Distance from base $= 6\sqrt{7}$ m

27. Question

An aeroplane leaves an airport and files due north at a speed of 1000 km/hr. At the same time, another aeroplane leaves the same airport and files due west at a speed of 1200 km/hr. How far apart will be the two planes after $1\frac{1}{2}$ hours?

Answer

Distance traveled by the plane flying towards north in 11/2 hrs

$= 1000 \times 1\frac{1}{2} = 1500$km

Similarly Distance traveled by the plane flying towards west in 11/2hrs

$= 1200 \times 1\frac{1}{2} = 1800$ km

Let this distance is represented by OA and OB

Distance between these place after $1\,{}^{1}/_{2}$hrs $AB = \sqrt{OA^2 + OB^2}$

$= \sqrt{\{1500\}2 + (1000)2} = \sqrt{2250000 + 3240000}$

$= \sqrt{5490000} = \sqrt{9x610000} = 300\sqrt{61}$

$= 300 \times 7.8102$

$= 2343.07$ km

So, distance between these places will be 2343 km (Approx) km, after 1 1/2 hrs

28. Question

Determine whether the triangle having sides (a – 1) cm, $2\sqrt{a}$ cm and (a + 1) cm is a right angled triangle.

Answer

Let ABC be the triangle

Where $AB=(a-1)^2$ cm

$BC=2\sqrt{a}$ cm

$CA=(a+1)$ cm

$AB^2=(a-1)^2=a^2+1-2a$

$BC^2=(2\sqrt{a})^2=4a^2$

$CA^2=(a+1)^2=a^2+1+2a$

Hence $AB^2+BC^2=AC^2$

SO ΔABC is a right angles triangle at B

CCE - Formative Assessment

1. Question

State basic proportionality theorem and its converse.

Answer

Basic Proportionality Theorem: If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

Converse of Basic Proportionality Theorem: If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

2. Question

In the adjoining figure, find AC.

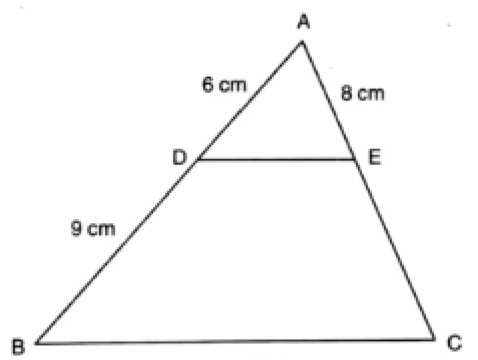


Fig. 4.234

Answer

From the given figure ΔABC, DE || BC.

Let EC = x cm.

We know that basic proportionality theorem states that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

Then $\frac{AD}{DB} = \frac{AE}{EC}$

$$\Rightarrow \frac{6}{9} = \frac{8}{x}$$

$$\Rightarrow x = \frac{8(9)}{6}$$

⇒ x = 12 cm = EC

Here, AC = AE + EC

⇒ AC = 8 + 12 = 20 cm

∴ AC = 20 cm

3. Question

In the adjoining figure, if AD is the bisector of ∠A, what is AC?

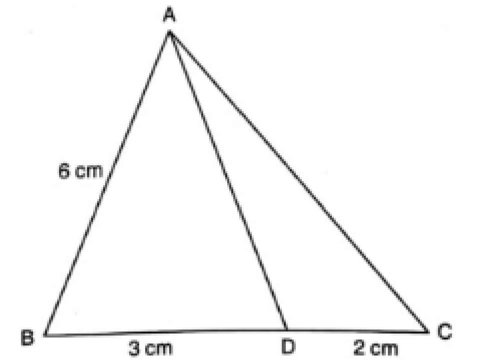


Fig. 4.235

Answer

Given AD is the bisector of ∠A in ΔABC. Let AC be x cm.

We know that the angle bisector theorem states that the internal bisector of an angle of a triangle divides the opposite side in two segments that are proportional to the other two sides of the triangle.

$$\Rightarrow \frac{AB}{AC} = \frac{DB}{DC}$$

$$\Rightarrow \frac{6}{x} = \frac{3}{2}$$

$$\Rightarrow x = \frac{6(2)}{3}$$

⇒ x = 4 cm

∴ AC = 4 cm

4. Question

State AAA similarity criterion.

Answer

AAA similarity criterion: In two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

5. Question

State SSS similarity criterion.

Answer

SSS similarity criterion: If in two triangles, sides of one triangle are proportional to the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.

6. Question

State SAS similarity criterion.

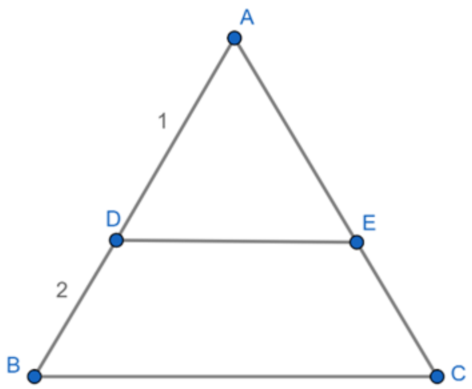
Answer

SAS similarity criterion: If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

7. Question

In the adjoining figure, DE is parallel to BC and AD = 1 cm, BD = 2 cm. What is the ratio of the area of A ABC to the area of A ADE?

Answer



Given DE || BC, AD = 1 cm and DB = 2 cm.

So, AB = 3 cm.

In ΔABC and ΔADE,

∠ABC = ∠ADE [corresponding angles]

∠ACB = ∠AED [corresponding angles]

∠A = ∠A [common angle]

We know that AAA similarity criterion states that in two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

∴ ΔABC ~ ΔADE

We know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

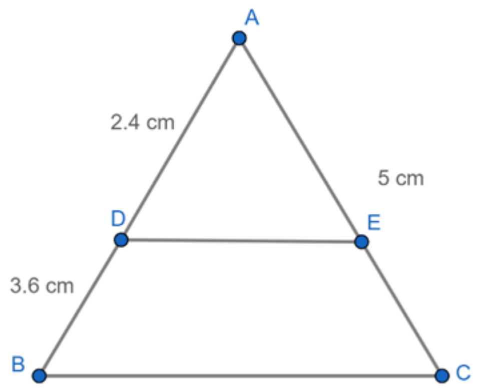
$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta ADE)} = \frac{(AB)^2}{(AD)^2} = \frac{3^2}{1^2} = \frac{9}{1}$$

∴ ar (ΔABC): ar (ΔADE) = 9: 1

8. Question

In the figure given below $DE \| BC$. If AD = 2.4 cm, DB = 3.6 cm and AC = 5 cm. Find AE.

Answer



Given DE || BC, AD = 2.4 cm, DB = 3.6 cm and AC = 5 cm.

We have to find AE.

We know that basic proportionality theorem states that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{AC - AE}$$

$$\Rightarrow \frac{2.4}{3.6} = \frac{AE}{5 - AE}$$

⇒ 2.4 (5 – AE) = 3.6 AE

⇒ 12 – 2.4 AE = 3.6 AE

⇒ 12 = 3.6 AE + 2.4 AE

⇒ 12 = 6 AE

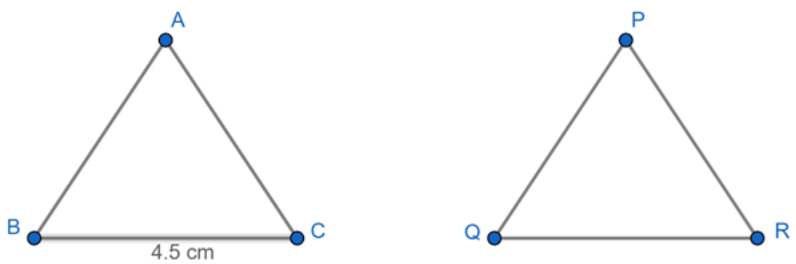
⇒ AE = 12/6

∴ AE = 2 cm

9. Question

If the areas of two similar triangles ABC and PQR are in the ratio 9 : 16 and BC = 4.5 cm, what is the length of QR?

Answer



Given ΔABC ~ ΔPQR, ar (ΔABC): ar (ΔPQR) = 9: 16 and BC = 4.5 cm

We know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{(BC)^2}{(QR)^2}$$

$$\Rightarrow \frac{9}{16} = \frac{(4.5)^2}{(QR)^2}$$

$$\Rightarrow QR^2 = \frac{20.25(16)}{9}$$

⇒ QR^2 = 2.25 (16)

⇒ QR^2 = 36

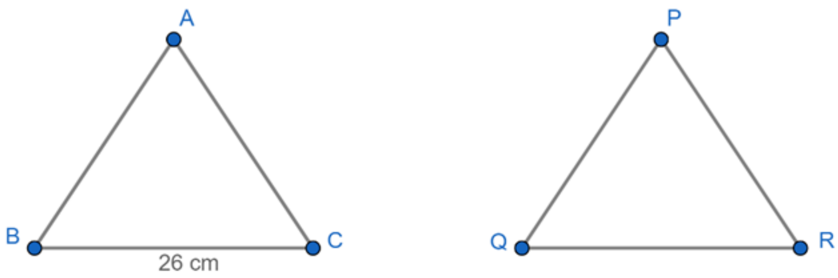
⇒ QR = 6

∴ The length of QR is 6 cm.

10. Question

The areas of two similar triangles are 169 cm^2 and 121 cm^2 respectively. If the longest side of the larger triangle is 26 cm, what is the length of the longest side of the smaller triangle?

Answer



Given ΔABC ~ ΔPQR, ar (ΔABC): ar (ΔPQR) = 169: 121 and BC = 26 cm

We know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{(BC)^2}{(QR)^2}$$

$$\Rightarrow \frac{169}{121} = \frac{(26)^2}{(QR)^2}$$

$$\Rightarrow QR^2 = \frac{26(26)(121)}{169}$$

⇒ $QR^2 = 4\ (121)$

⇒ $QR^2 = 484$

⇒ QR = 22

∴ The length of QR is 22 cm.

11. Question

If ABC and DEF are similar triangles such that $\angle A$ = 57° and $\angle E$ = 73°, what is the measure of $\angle C$?

Answer

Given ABC and DEF are two similar triangles, ∠A = 57° and ∠E = 73°

We know that SAS similarity criterion states that if one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

In ΔABC and ΔDEF,

if $\frac{AB}{DE} = \frac{AC}{DF}$ and ∠A = ∠D, then ΔABC ~ ΔDEF

So, ∠A = ∠D

⇒ ∠D = 57° … (1)

Similarly, ∠B = ∠E

⇒ ∠B = 73° … (2)

We know that the sum of all angles of a triangle is equal to 180°.

⇒ ∠A + ∠B + ∠C = 180°

⇒ 57° + 73° + ∠C = 180°

⇒ 130° + ∠C = 180°

⇒ ∠C = 180° - 130° = 50°

∴ ∠C = 50°

12. Question

If the altitude of two similar triangles are in the ratio 2 : 3, what is the ratio of their areas?

Answer

Given altitudes of two similar triangles are in ratio 2: 3.

Let first triangle be ΔABC and second triangle be ΔPQR.

We know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{(2)^2}{(3)^2}$$

∴ ar (ΔABC): ar (ΔPQR) = 4: 9

13. Question

If ΔABC and ΔDEF are two triangles such that $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = \frac{3}{4}$, then write Area (ΔABC): Area (ΔDEF).

Answer

Given that ΔABC and ΔDEF are two triangles such that $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = \frac{3}{4}$

Here, the corresponding sides are given proportional.

We know that two triangles are similar if their corresponding sides are proportional.

And we know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{(3)^2}{(4)^2}$$

∴ Area (ΔABC): Area (ΔDEF) = 9: 16

14. Question

If ΔABC and ΔDEF are similar triangles such that AB = 3 cm, BC = 2 cm CA = 2.5 cm and EF = 4 cm, write the perimeter of ΔDEF .

Answer

Given that ΔABC and ΔDEF are similar triangles such that AB = 3 cm, BC = 2 cm, CA = 2.5 cm and EF = 4 cm.

We know that two triangles are similar if their corresponding sides are proportional.

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$

First consider,

$$\frac{AB}{BC} = \frac{DE}{EF}$$

$$\Rightarrow \frac{3}{2} = \frac{DE}{4}$$

⇒ DE = 6 cm … (1)

Now,

$$\frac{CA}{BC} = \frac{DF}{EF}$$

$$\Rightarrow \frac{2.5}{2} = \frac{DF}{4}$$

⇒ DF = 5 cm … (2)

Then, perimeter of ΔDEF = DE + EF + DF = 6 + 4 + 5

∴ Perimeter of ΔDEF = 15 cm

15. Question

State Pythagoras theorem and its converse.

Answer

Pythagoras Theorem: In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Converse of Pythagoras Theorem: In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to first side is a right angle.

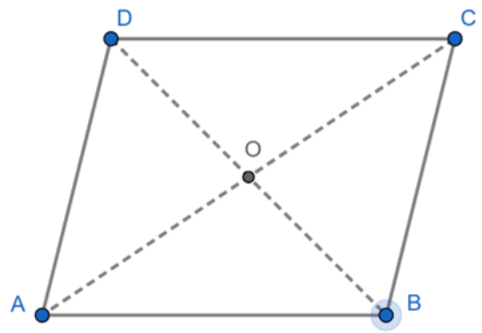
16. Question

The lengths of the diagonals of a rhombus are 30 cm and 40 cm. Find the side of the rhombus. [CBSE 2008]

Answer

Given the lengths of the diagonals of a rhombus are 30 cm and 40 cm.

Let the diagonals AC and BD of the rhombus ABCD meet at point O.



We know that the diagonals of the rhombus bisect each other perpendicularly.

Also we know that Pythagoras theorem states that in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Consider right triangle AOD,

$\Rightarrow AD^2 = AO^2 + OD^2$

$= 15^2 + 20^2$

$= 225 + 400$

$= 625$

$\Rightarrow AD = 25$ cm

∴ The side of the rhombus is 25 cm.

17. Question

In Fig. 4.236, $PQ \| BC$ and AP : PB = 1 : 2. Find $\frac{area\ (\Delta APQ)}{area\ (\Delta ABC)}$ [CBSE 2008]

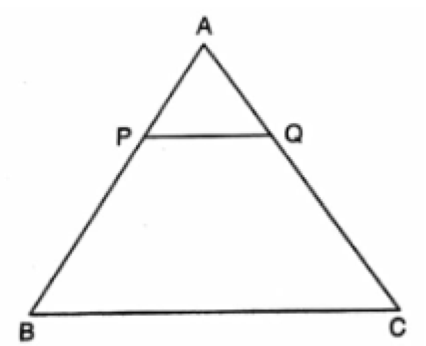


Fig. 4.236

Answer

Given in the given figure PQ || BC and AP: PB = 1: 2

We know that basic proportionality theorem states that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

Since Δ APQ and ΔABC are similar, $\frac{AP}{AB} = \frac{AQ}{AC} = \frac{PQ}{BC}$

Given $\frac{AP}{PB} = \frac{1}{2}$

⇒ PB = 2AP

So, $\frac{AP}{AB} = \frac{AP}{AP+PB} = \frac{AP}{AP+2AP} = \frac{1}{3}$

we know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{(AP)^2}{(AB)^2}$$

$$= \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

∴ Area (ΔAPB): Area (ΔABC) = 1: 9

18. Question

In Fig. 4.237, LM = LN = 46°. Express x in terms of a, b and c where a, b, c are lengths of LM, MN and and NK respectively.

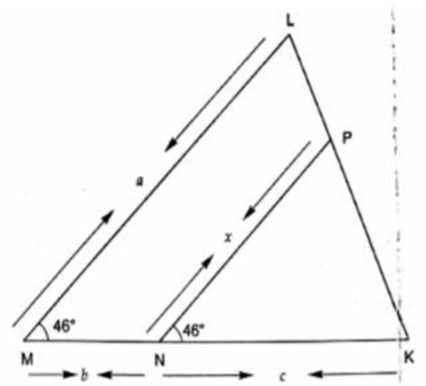


Fig. 4.237

Answer

Given ∠M = ∠N = 46°

It forms a pair of corresponding angles, hence LM || PN.

In ΔLMK and ΔPNK,

∠LMK = ∠PNK [corresponding angles]

∠MLK = ∠NPK [corresponding angles]

∠K = ∠K [common angle]

We know that AAA similarity criterion states that in two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

∴ ΔLMK ~ ΔPNK

We know that two triangles are similar if their corresponding sides are proportional.

$$\Rightarrow \frac{ML}{NP} = \frac{MK}{NK}$$

$$\Rightarrow \frac{a}{x} = \frac{b+c}{c}$$

$$\therefore x = \frac{ac}{b+c}$$

19. Question

In Fig. 4.238, S and T are points on the sides PQ and PR respectively of A PQR such that PT = 2 cm, TR = 4 cm and ST is parallel to QR. Find the ratio of the areas of ΔPST and ΔPQR. [CBSE 2010]

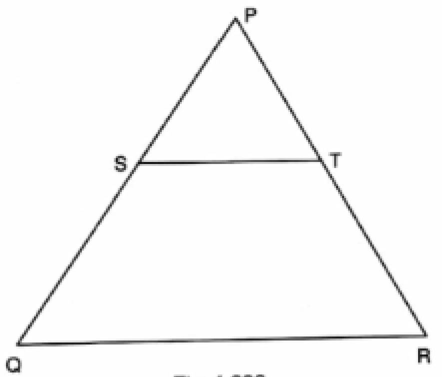


Fig. 4.238

Answer

Given ST || QR, TR = 4 cm and PT = 2 cm.

So, PR = 6 cm.

In ΔPST and ΔPQR,

∠PST = ∠PQR [corresponding angles]

∠PTS = ∠PRQ [corresponding angles]

∠P = ∠P [common angle]

We know that AAA similarity criterion states that in two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

∴ ΔPST ~ ΔPQR

We know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \frac{ar(\Delta PST)}{ar(\Delta PQR)} = \frac{(PT)^2}{(PR)^2} = \frac{2^2}{6^2} = \frac{4}{36} = \frac{1}{9}$$

∴ ar (ΔPST): ar (ΔPQR) = 1: 9

20. Question

In Fig. 4.239, ΔAHK is similar to ΔABC. If AK = 10 cm, BC = 3.5 cm and HK = 7 cm, find AC. [CBSE 2010]

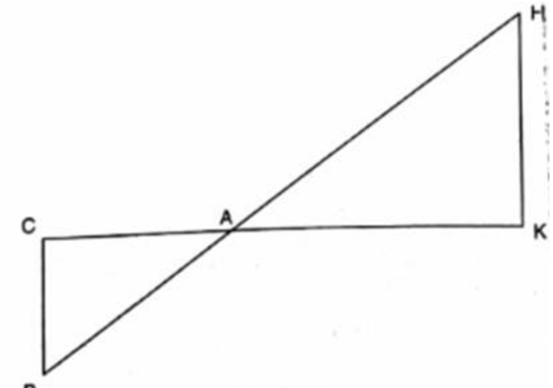


Fig. 4.239

Answer

Given ΔAHK ~ ΔABC, AK = 10 cm, BC = 3.5 cm and HK = 7 cm.

We know that two triangles are similar if their corresponding sides are proportional.

$$\Rightarrow \frac{AC}{AK} = \frac{BC}{HK}$$

$$\Rightarrow \frac{AC}{10} = \frac{3.5}{7}$$

∴ AC = 5 cm

21. Question

In Fig. 4.240, $DE \| BC$ in ΔABC such that BC = 8 cm, AB = 6 cm and DA =1.5 cm. Find DE.

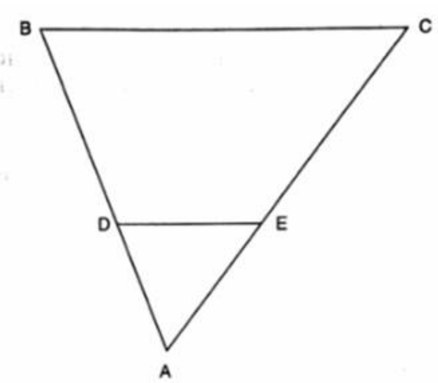


Fig. 4.240

Answer

Given DE || BC, BC = 8 cm, AB = 6 cm and DA = 1.5 cm.

So, PR = 6 cm.

In ΔABC and ΔADE,

∠ABC = ∠ADE [corresponding angles]

∠ACB = ∠AED [corresponding angles]

∠A = ∠A [common angle]

We know that AAA similarity criterion states that in two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

∴ ΔABC ~ ΔADE

We know that two triangles are similar if their corresponding sides are proportional.

$$\Rightarrow \frac{BC}{DE} = \frac{AB}{DA}$$

$$\Rightarrow \frac{8}{DE} = \frac{6}{1.5}$$

∴ DE = 2 cm

22. Question

In Fig. 4.241, $DE \| BC$ and AD = $\frac{1}{2}$ BD. If BC = 4.5 cm, find DE.

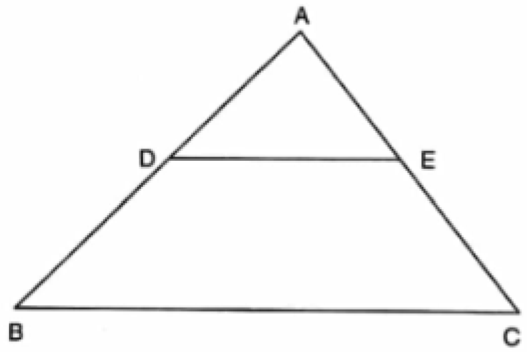


Fig. 4.241

Answer

Given DE || BC, AD = 1/2 BD and BC = 4.5 cm

In ΔABC and ΔADE,

∠ABC = ∠ADE [corresponding angles]

∠ACB = ∠AED [corresponding angles]

∠A = ∠A [common angle]

We know that AAA similarity criterion states that in two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

∴ ΔABC ~ ΔADE

We know that two triangles are similar if their corresponding sides are proportional.

$$\Rightarrow \frac{AD}{AB} = \frac{DE}{BC}$$

$$\Rightarrow \frac{AD}{AD + BD} = \frac{DE}{BC}$$

$$\Rightarrow \frac{\frac{1}{2}BD}{\frac{1}{2}BD + BD} = \frac{DE}{BC}$$

$$\Rightarrow \frac{1}{3} = \frac{DE}{BC}$$

$$\Rightarrow \frac{1}{3} = \frac{DE}{4.5}$$

∴ DE = 1.5 cm

1. Question

A vertical stick 20 m long casts a shadow 10 m long on the ground. At the same time, a tower casts a shadow 50 m long on the ground. The height of the tower is

A. 100 m

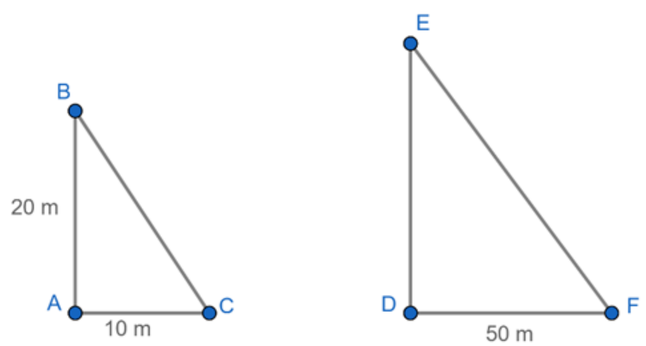
B. 120 m

C. 25 m

D. 200 m.

Answer

Given A vertical stick 20 m long casts a shadow 10 m long on the ground and a tower casts a shadow 50 m long on the ground.



We know that AAA similarity criterion states that in two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

In ΔABC and ΔDEF,

∠A = ∠D = 90°, ∠C = ∠F

∴ ΔABC ~ ΔDEF

We know that if two triangles are similar then their sides are proportional.

$$\Rightarrow \frac{AB}{DE} = \frac{AC}{DF}$$

$$\Rightarrow \frac{20}{DE} = \frac{10}{50}$$

∴ DE = 100 m

2. Question

Sides of two similar triangles are in the ratio 4 : 9 . Areas of these triangles are in the ratio.

A. 2 : 3

B. 4 : 9

C. 81 : 16

D. 16 : 81

Answer

Given sides of two similar triangles are in the ratio 4: 9.

We know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \frac{ar(\Delta 1)}{ar(\Delta 2)} = \frac{(side1)^2}{(side2)^2} = \frac{4^2}{9^2} = \frac{16}{81}$$

∴ ar (Δ1): ar (Δ2) = 16: 81

3. Question

The areas of two similar triangles are in respectively 9 cm^2 and 16 cm^2. The ratio of their corresponding sides is

A. 3:4

B. 4 : 3

C. 2 : 3

D. 4 : 5

Answer

Given that area of two similar triangles are 9 cm^2 and 16 cm^2.

We know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \frac{ar(\Delta 1)}{ar(\Delta 2)} = \frac{(side1)^2}{(side2)^2}$$

$$\Rightarrow \frac{9}{16} = \frac{(side1)^2}{(side2)^2}$$

$$\Rightarrow \frac{side1}{side2} = \frac{3}{4}$$

∴ Ratio of their corresponding sides is 3: 4.

4. Question

The areas of two similar triangles ΔABC and ΔDEF are 144 cm^2 and 81 cm^2 respectively. If the longest side of larger A ABC be 36 cm, then. the longest side of the smaller triangle ΔDEF is

A. 20 cm

B. 26 cm

C. 27 cm

D. 30 cm

Answer

Given that area of two similar triangles ΔABC and ΔDEF are 144 cm^2 and 81 cm^2. Also the longest side of larger ΔABC is 36 cm.

We have to find the longest side of the smaller triangle ΔDEF. Let it be x.

We know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{(longest\ side\ of\ \Delta ABC)^2}{(longest\ side\ of\ \Delta DEF)^2}$$

$$\Rightarrow \frac{144}{81} = \frac{(36)^2}{(x)^2}$$

$$\Rightarrow \frac{36}{x} = \frac{12}{9}$$

⇒ x = 27 cm

∴ Longest side of ΔDEF is 27 cm.

5. Question

ΔABC and ΔBDE are two equilateral triangles such that D is the mid-point of BC. The ratio of the areas of triangles ABC and BDE is

A. 2 : 1

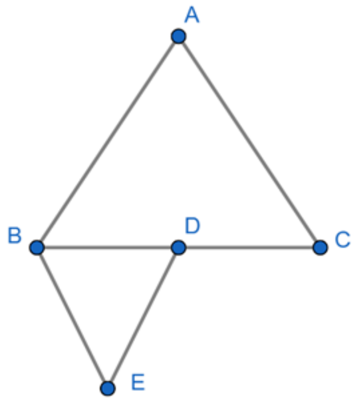
B. 1 :2

C. 4 : 1

D. 1 : 4

Answer

Given ΔABC and ΔBDE are two equilateral triangles such that D is the midpoint of BC.



Since the given triangles are equilateral, they are similar triangles.

And also since D is the mid-point of BC, BD = DC.

We know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta BDE)} = \frac{(BC)^2}{(BD)^2}$$

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta BDE)} = \frac{(BD + DC)^2}{(BD)^2}$$

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta BDE)} = \frac{(BD + BD)^2}{(BD)^2}$$

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta BDE)} = \frac{(2BD)^2}{(BD)^2}$$

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta BDE)} = \frac{4}{1}$$

∴ ar (ΔABC): ar (ΔBDE) = 4: 1

6. Question

Two isosceles triangles have equal angles and their areas are in the ratio 16 : 25. The ratio of their corresponding heights is

A. 4 : 5

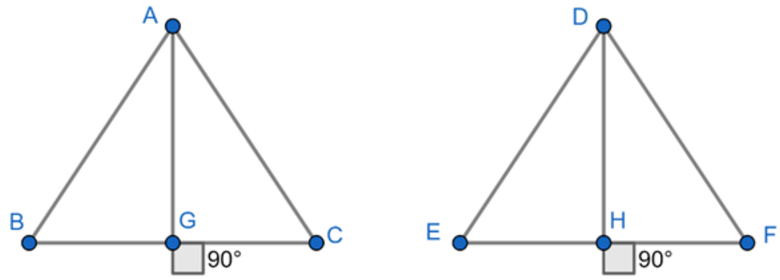
B. 5 : 4

C. 3 : 2

D. 5 : 7

Answer

Given two isosceles triangles have equal angles and their areas are in the ratio 16 : 25.



We know that SAS similarity criterion states that if one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

In ΔABC and ΔDEF,

if $\frac{AB}{DE} = \frac{AC}{DF}$ and ∠A = ∠D, then ΔABC ~ ΔDEF

We know that the ratio of areas of two similar triangles is equal to the ratio of the squares of their corresponding altitudes.

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AG}{DH}\right)^2$$

$$\Rightarrow \frac{16}{25} = \left(\frac{AG}{DH}\right)^2$$

$$\Rightarrow \frac{AG}{DH} = \frac{4}{5}$$

∴ AG: DH = 4: 5

7. Question

If ΔABC and ΔDEF are similar such that 2 AB = DE and BC = 8 cm, then EF =

A. 16 cm

B. 12 cm

C. 8 cm

D. 4 cm.

Answer

Given ΔABC and ΔDEF are similar triangles such that 2AB = DE and BC = 8 cm

We know that if two triangles are similar then their sides are proportional.

For ΔABC and ΔDEF,

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF}$$

$$\Rightarrow \frac{1}{2} = \frac{8}{EF}$$

∴ EF = 16 cm

8. Question

If ΔABC and ΔDEF are two triangles such that $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = \frac{2}{5}$, then Area (ΔABC): Area (ΔDEF) =

A. 2 : 5

B. 4 : 25

C. 4 : 15

D. 8 : 125

Answer

Given ΔABC and ΔDEF are two triangles such that $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = \frac{2}{5}$

We know that if two triangles are similar then their sides are proportional.

Since $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$, ΔABC and ΔDEF are similar.

We know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{(AB)^2}{(DE)^2}$$

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{(2)^2}{(5)^2}$$

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{4}{25}$$

∴ ar (ΔABC): ar (ΔDEF) = 4: 25

9. Question

ΔABC is such that AB = 3 cm, BC = 2 cm and CA = 2 . 5 cm. If $\Delta DEF \sim \Delta ABC$ and EF = 4 cm, then perimeter of ΔDEF is

A. 7.5 cm

B. 15 cm

C. 22.5 cm

D. 30 cm.

Answer

Given that ΔABC and ΔDEF are similar triangles such that AB = 3 cm, BC = 2 cm, CA = 2.5 cm and EF = 4 cm.

We know that two triangles are similar if their corresponding sides are proportional.

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$

First consider,

$$\frac{AB}{BC} = \frac{DE}{EF}$$

$$\Rightarrow \frac{3}{2} = \frac{DE}{4}$$

⇒ DE = 6 cm … (1)

Now,

$$\frac{CA}{BC} = \frac{DF}{EF}$$

$$\Rightarrow \frac{2.5}{2} = \frac{DF}{4}$$

⇒ DF = 5 cm … (2)

Then, perimeter of ΔDEF = DE + EF + DF = 6 + 4 + 5

∴ Perimeter of ΔDEF = 15 cm

10. Question

XY is drawn parallel to the base BC of ΔABC cutting AB at X and AC at Y. If AB = 4 BX and YC = 2 cm, then AY =

A. 2 cm

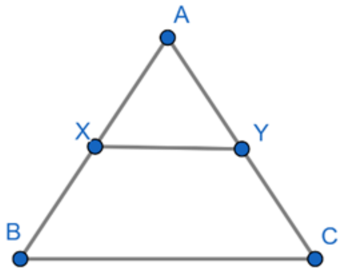
B. 4 cm

C. 6 cm

D. 8 cm.

Answer

Given XY is drawn parallel to the base BC of a ΔABC cutting AB at X and AC at Y. AB = 4BX and YC = 2 cm.



In ΔAXY and ΔABC,

∠AXY = ∠ABC [corresponding angles]

∠AYX = ∠ACB [corresponding angles]

∠A = ∠A [common angle]

We know that AAA similarity criterion states that in two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

∴ ΔAXY ~ ΔABC

Let BX = x, so AB = 4x and AX = 3x.

We know that two triangles are similar if their corresponding sides are proportional.

$$\Rightarrow \frac{AX}{BX} = \frac{AY}{YC}$$

$$\Rightarrow \frac{3x}{x} = \frac{AY}{2}$$

∴ AY = 6 cm

11. Question

Two poles of height 6 m and 11 m stand vertically upright on a plane ground. If the distance between their foot is 12 m, the distance between their tops is

A. 12 m

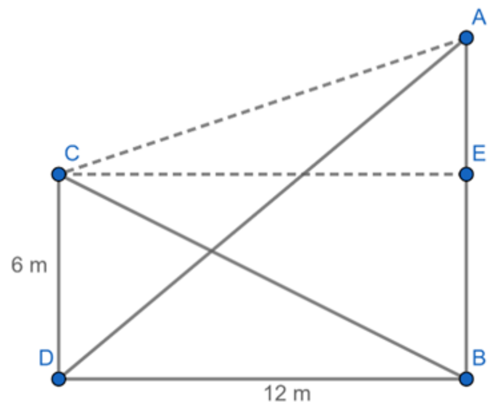
B. 14 m

C. 13 m.

D. 11 m

Answer

Given two poles of heights 6 m and 11 m stand vertically upright on a plane ground. Distance between their foot is 12 m.



Let CD be the pole with height 6 m. AB is the pole with height 11m and DB = 12 m

Let us assume a point E on the pole AB which is 6m from the base of AB.

Hence AE = AB − 6 = 11 − 6 = 5m

We know that the Pythagoras theorem state that in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Now, in right triangle AEC,

⇒ $AC^2 = AE^2 + EC^2$

Since CDEB forms a rectangle and opposite sides of rectangle are equal,

⇒ $AC^2 = 5^2 + 12^2$

= 25 + 144

= 169

⇒ AC = 13

∴ The distance between their tops is 13 m.

12. Question

In ΔABC, a line XY parallel to BC cuts AB at X and AC at Y. If BY bisects ΔXYC, then

A. BC =CY

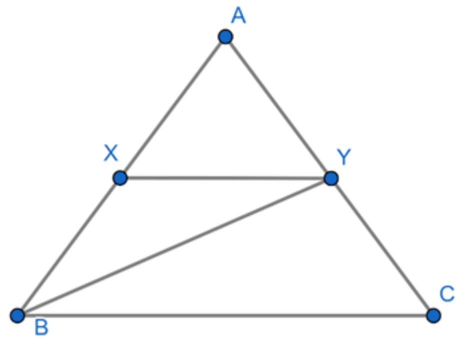
B. BC = BY

C. BC $\neq$ CY

D. BC $\neq$ BY

Answer

Given in ΔABC, XY || BC and BY is a bisector of ∠XYC.



Since XY || BC,

∠YBC = ∠BYC [alternate angles]

Now, in Δ BYC, two angles are equal.

Hence, two corresponding sides will be equal.

∴ BC = CY

13. Question

In ΔABC, D and E are points on side AB and AC respectively such that $DE \| BC$ and AD: DB = 3 : 1. If EA = 3.3 cm, then AC =

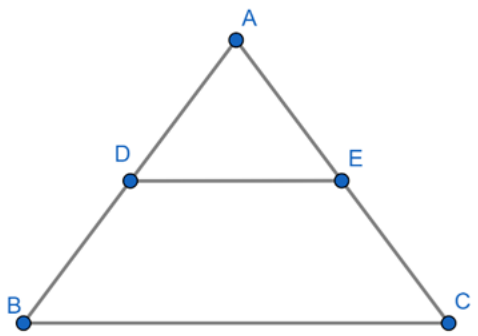
A. 1.1 cm

B. 4 cm

C. 4.4 cm

D. 5.5 cm

Answer



From the given figure ΔABC, DE || BC.

Let AC = x cm.

We know that basic proportionality theorem states that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

Then $\frac{AD}{AB} = \frac{AE}{AC}$

$$\Rightarrow \frac{AD}{AD + BD} = \frac{3.3}{x}$$

$$\Rightarrow \frac{AD}{AD + \frac{1}{3}AD} = \frac{3.3}{x}$$

⇒ x = 4.4 cm

∴ AC = 4.4 cm

14. Question

In triangles ABC and DEF, $\angle A = \angle E = 40°$, AB : ED = AC : EF and $\angle F$ = 65°, then $\angle B$ =

A. 35°

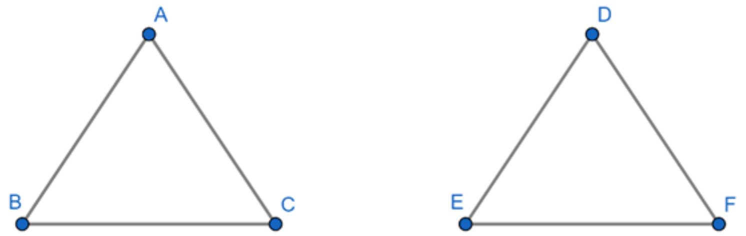
B. 65°

C. 75°

D. 85°

Answer

Given in triangles ABC and DEF, ∠A = ∠E = 40°, AB: ED = AC: EF and ∠F = 65°.



We know that SAS similarity criterion states that if one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

In ΔABC and ΔDEF,

∠A = ∠E and AB: ED = AC: EF then ΔABC ~ ΔDEF

So, ∠A = ∠E = 40°

⇒ ∠C = ∠F = 65°

Similarly, ∠B = ∠D

We know that the sum of all angles of a triangle is equal to 180°.

⇒ ∠A + ∠B + ∠C = 180°

⇒ 40° + ∠B + 65° = 180°

⇒ ∠B + 115° = 180°

⇒ ∠B = 180° - 115° = 75°

∴ ∠B = 75°

15. Question

If ABC and DEF are similar triangles such that $\angle A$ = 47° and $\angle E$ = 83°, then $\angle C$ =

A. 50°

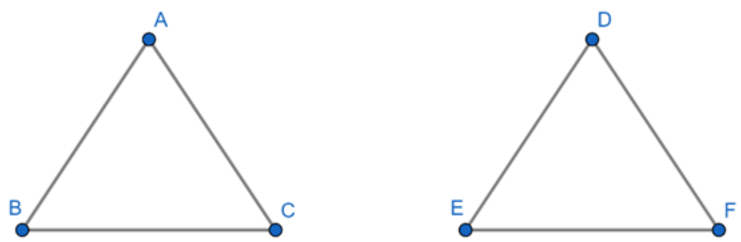
B. 60°

C. 70°

D. 80°

Answer

Given ABC and DEF are two similar triangles, ∠A = 47° and ∠E = 83°



We know that SAS similarity criterion states that if one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

In ΔABC and ΔDEF,

if $\frac{AB}{DE} = \frac{AC}{DF}$ and ∠A = ∠D, then ΔABC ~ ΔDEF

So, ∠A = ∠D

⇒ ∠D = 47° … (1)

Similarly, ∠B = ∠E

⇒ ∠B = 83° … (2)

We know that the sum of all angles of a triangle is equal to 180°.

⇒ ∠A + ∠B + ∠C = 180°

⇒ 47° + 83° + ∠C = 180°

⇒ 130° + ∠C = 180°

⇒ ∠C = 180° - 130° = 50°

∴ ∠C = 50°

16. Question

If D, E, F are the mid-points of sides BC, CA and AB respectively of A ABC, then the ratio of the areas of triangles DEF and ABC is

A. 1 :4

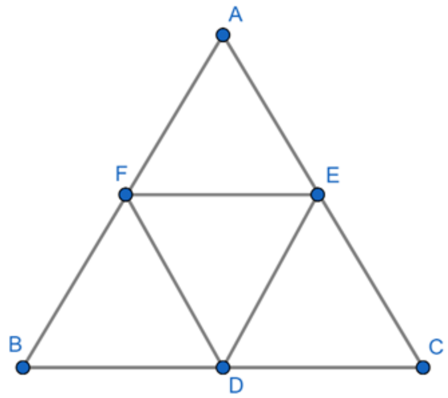
B. 1 : 2

C. 2 : 3

D. 4 : 5

Answer

Given D, E and F are the mid-points of sides BC, CA and AB respectively of ΔABC.



Then DE || AB, DE || FA … (1)

And DF || CA, DF || AE … (2)

From (1) and (2), we get AFDE is a parallelogram.

Similarly, BDEF is a parallelogram.

In ΔADE and ΔABC,

⇒ ∠FDE = ∠A [Opposite angles of ||gm AFDE]

⇒ ∠DEF = ∠B [Opposite angles of ||gm BDEF]

∴ By AA similarity criterion, ΔABC ~ ΔDEF.

We know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{(DE)^2}{(AB)^2}$$

$$\Rightarrow \frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{\left(\frac{1}{2}AB\right)^2}{(AB)^2}$$

$$\Rightarrow \frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{1}{4}$$

∴ ar (ΔDEF): ar (ΔABC) = 1: 4

17. Question

In a ΔABC, $\angle A$ = 90°, AB = 5 cm and AC = 12 cm. If $AD \perp BC$, then AD =

A. $\frac{13}{2}$ cm

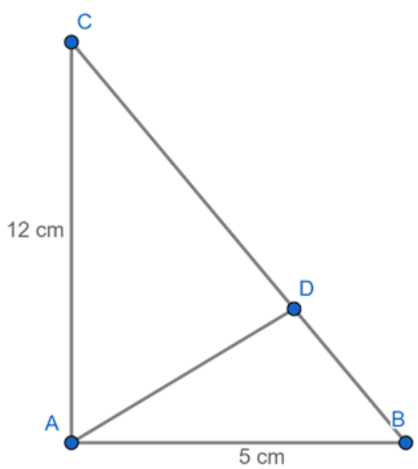
B. $\frac{60}{13}$ cm

C. $\frac{13}{60}$ cm

D. $\frac{2\sqrt{15}}{13}$ cm 13

Answer

Given in ΔABC, ∠A = 90°, AB = 5 cm, AC = 12 cm and AD ⊥ BC



In ΔACB and ΔADC,

∠CAB = ∠ADC [90°]

∠ABC = ∠CAD [corresponding angles]

∠C = ∠C [common angle]

We know that AAA similarity criterion states that in two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

∴ ΔACB ~ ΔADC

$$\Rightarrow \frac{AD}{AB} = \frac{AC}{BC}$$

$$\Rightarrow AD = \frac{AB(AC)}{BC}$$

$$\Rightarrow AD = \frac{12(5)}{13}$$

$$\Rightarrow AD = \frac{60}{13}$$

∴ AD = 60/13 cm

18. Question

In an equilateral triangle ABC, if $AD \perp BC$, then

A. $2AB^2 = 3AD^2$

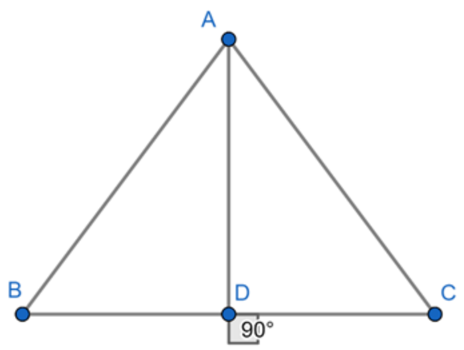
B. $4AB^2 = 3\ AD^2$

C. $3AB^2 = 4AD2$

D. $3AB^2 = 2AD^2$

Answer

Given in equilateral ΔABC, AD ⊥ BC.



We know that the Pythagoras theorem state that in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

In ΔABD,

$\Rightarrow AB^2 = AD^2 + BD^2$

$\Rightarrow AB^2 = AD^2 + (1/2BC)^2$ [∵ BD = 1/2BC]

$\Rightarrow AB^2 = AD^2 + (1/2AB)^2$ [∵ AB = BC]

$\Rightarrow AB^2 = AD^2 + 1/4AB^2$

∴ $3AB^2 = 4AD^2$

19. Question

If ΔABC is an equilateral triangle such that $AD \perp BC$, then AD^2 =

A. $\frac{3}{2}$ DC^2

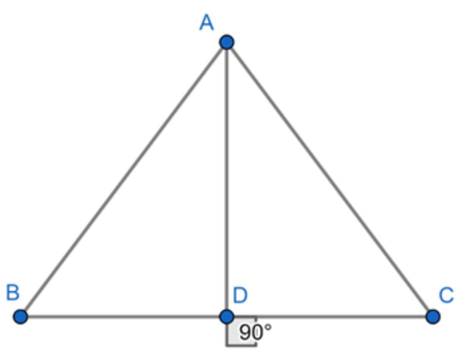
B. 2 DC^2

C. 3 CD^2

D. 4 DC^2

Answer

Given in an equilateral ΔABC, AD ⊥ BC



Since AD ⊥ BC, BD = CD = BC/2

We know that the Pythagoras theorem state that in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Now, in right triangle ADC,

⇒ $AC^2 = AD^2 + DC^2$

⇒ $BC^2 = AD^2 + DC^2$

⇒ $(2DC)^2 = AD^2 + DC^2$

⇒ $4DC^2 = AD^2 + DC^2$

⇒ $3DC^2 = AD^2$

∴ $3CD^2 = AD^2$

20. Question

In a ΔABC, perpendicular AD from A on BC meets BC at D. If BD = 8 cm, DC = 2 cm and AD = 4 cm, then

A. ΔABC is isosceles

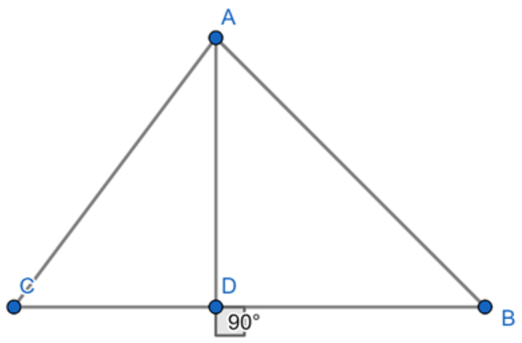
B. ΔABC is equilateral

C. AC = 2 AB

D. ΔABC is right-angled at A.

Answer

Given in ΔABC, AD ⊥ BC, BD = 8 cm, DC = 2 cm and AD = 4 cm.



We know that the Pythagoras theorem state that in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Now, in right triangle ADC,

⇒ $AC^2 = AD^2 + DC^2$

⇒ $AC^2 = (4)^2 + (2)^2$

= 16 + 4

∴ $AC^2 = 20$ … (1)

In ΔADB,

⇒ $AB^2 = AD^2 + BD^2 = 4^2 + 8^2 = 16 + 64$

∴ $AB^2 = 80$ … (2)

Now, in ΔABC,

⇒ $BC^2 = (CD + DB)^2 = (2 + 8)^2 = 10^2 = 100$

And $AB^2 + CA^2 = 80 + 20 = 100$

∴ $AB^2 + CA^2 = BC^2$

Hence, ΔABC is right angled at A.

21. Question

In a ΔABC, point D is on side AB and point E is on side AC, such that BCED is a trapezium. If DE : BC = 3 : 5, then Area (ΔADE): Area (âBCED) =

A. 3 : 4

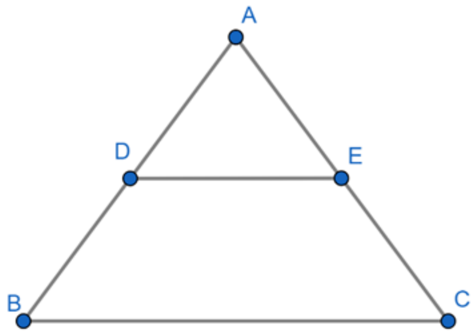
B. 9: 16

C. 3: 5

D. 9 : 25

Answer

Given in ΔABC, point D is on side AB and point E is on side AC, such that BCED is a trapezium and DE: BC = 3: 5.



In ΔABC and ΔADE,

∠ABC = ∠ADE [corresponding angles]

∠ACB = ∠AED [corresponding angles]

∠A = ∠A [common angle]

We know that AAA similarity criterion states that in two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

∴ ΔABC ~ ΔADE

We know that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{(DE)^2}{(BC)^2} = \frac{3^2}{5^2} = \frac{9}{25}$$

Let ar (ΔADE) = 9x sq. units and ar (ΔABC) = 25x sq. units

⇒ ar (trap BCED) = ar (ΔABC) – ar (ΔADE)

= 25x – 9x

= 16x sq. units

Now,

$$\Rightarrow \frac{ar(\Delta ADE)}{ar(trap\ BCED)} = \frac{9x}{16x} = \frac{9}{16}$$

∴ ar (ΔADE): ar (trap BCED) = 9: 16

22. Question

In a ΔABC, AD is the bisector of $\angle BAC$. If AB = 6 cm, AC = 5 cm and BD = 3 cm„ then DC =

A. 11.3 cm

B. 2.5 cm

C. 3 5 cm

D. None of these.

Answer

Given AD is the bisector of ∠BAC. AB = 6 cm, AC = 5 cm and BD = 3 cm.

We know that the internal bisector of angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$$\Rightarrow \frac{AB}{AC} = \frac{BD}{DC}$$

$$\Rightarrow \frac{6}{5} = \frac{3}{DC}$$

∴ DC = 2.5 cm

23. Question

In a ΔABC, AD is the bisector of $\angle BAC$. If AB = 8 cm, BD = 6 cm and DC = 3 cm. Find AC

A. 4 cm

B. 6 cm

C. 3 cm

D. 8 cm

Answer

Given AD is the bisector of ∠BAC. AB = 8 cm, DC = 3 cm and BD = 6 cm.

We know that the internal bisector of angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$$\Rightarrow \frac{AB}{AC} = \frac{BD}{DC}$$

$$\Rightarrow \frac{8}{AC} = \frac{6}{3}$$

∴ AC = 4 cm

24. Question

ABCD is a trapezium such that $BC \| AD$ and AB = 4 cm. If the diagonals AC and BD intersect at O such that $\frac{AO}{OC} = \frac{DO}{OB} = \frac{1}{2}$, then BC =

A. 7 cm

B. 8 cm

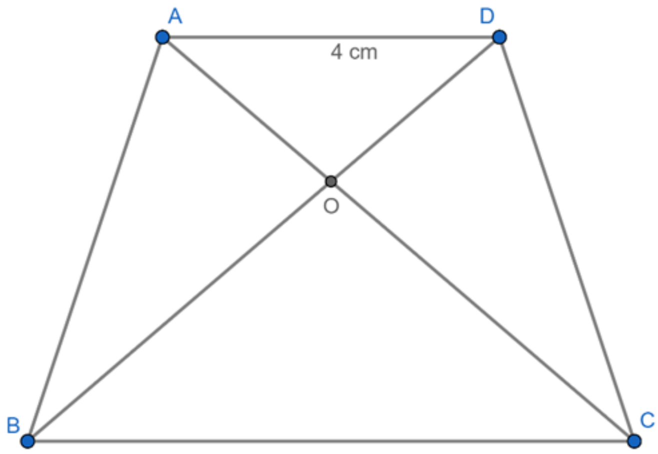
C. 9 cm

D. 6 cm

Answer

Given ABCD is a trapezium in which BC || AD and AD = 4 cm.

Also, the diagonals AC and BD intersect at O such that $\frac{AO}{OC} = \frac{DO}{OB} = \frac{1}{2}$



In ΔAOD and ΔCOB,

∠OAD = ∠OCB [alternate angles]

∠ODA = ∠OBC [alternate angles]

∠AOD = ∠BOC [vertically opposite angles]

We know that AAA similarity criterion states that in two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

∴ ΔAOD ~ ΔCOB

We know that two triangles are similar if their corresponding sides are proportional.

$$\Rightarrow \frac{AO}{CO} = \frac{DO}{BO} = \frac{AD}{BC}$$

$$\Rightarrow \frac{1}{2} = \frac{AD}{BC}$$

$$\Rightarrow \frac{1}{2} = \frac{4}{BC}$$

∴ BC = 8 cm

25. Question

If ABC is an isosceles triangle and D is a point on BC such that $AD \perp BC$, then

A. $AB^2 - AD^2 = BD.DC$

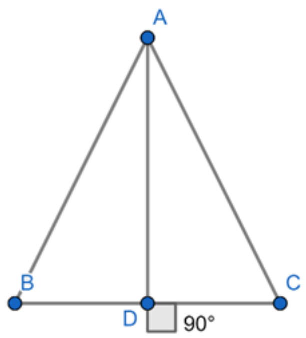
B. $AB^2 - AD^2 = BD^2 - DC^2$

C. $AB^2 + AD^2 = BD.DC$

D. $AB^2 + AD^2 = BD^2 - DC^2$

Answer

Given ABC is an isosceles triangles and AD ⊥ BC.



We know that in an isosceles triangle, the perpendicular from the vertex bisects the base.

∴ BD = DC

We know that the Pythagoras theorem state that in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Now, in right triangle ABD,

$\Rightarrow AB^2 = AD^2 + BD^2$

$\Rightarrow AB^2 - AD^2 = BD^2$

$\Rightarrow AB^2 - AD^2 = BD\,(BD)$

Since BD = DC,

$\therefore AB^2 - AD^2 = BD\,(DC)$

26. Question

ΔABC is a right triangle right-angled at A and $AD \perp BC$. Then, $\frac{BD}{DC}$ =

A. $\left(\frac{AB}{AC}\right)^2$

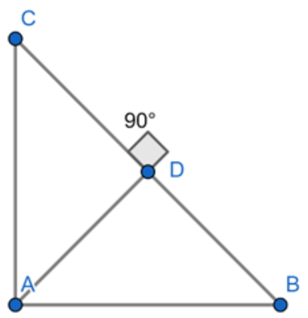
B. $\frac{AB}{AC}$

C. $\left(\frac{AB}{AD}\right)^2$

D. $\frac{AB}{AD}$

Answer

Given ΔABC is a right triangle right-angled at A and AD ⊥ BC.



⇒ ∠CAD + ∠BAD = 90° … (1)

⇒ ∠BAD + ∠ABD = 90° … (2)

From (1) and (2),

∠CAD = ∠ABD

By AA similarity,

In ΔADB and ΔADC,

⇒ ∠ADB = ∠ADC [90° each]

⇒ ∠ABD = ∠CAD

∴ ΔADB ~ ΔADC

We know that if two triangles are similar, their corresponding angles are equal and corresponding sides are proportional.

$$\therefore \frac{BD}{DC} = \frac{AB}{AC}$$

27. Question

If ABC is a right triangle right-angled at B and M, N are the mid-points of AB and BC respectively, then 4 (AN^2 + CM^2) =

A. 4 AC^2

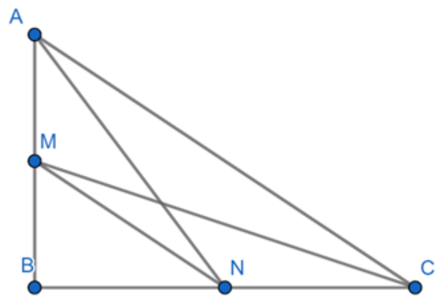
B. 5 AC^2

C. $\frac{5}{4}$ AC^2

D. 6 AC^2

Answer

Given ABC is a right triangle right-angled at B and M, N are mid-points of AB and BC respectively.



M is the mid-point of AB.

$$\Rightarrow BM = \frac{AB}{2}$$

And N is the mid-point of BC.

$$\Rightarrow BN = \frac{BC}{2}$$

Now,

⇒ $AN^2 + CM^2 = (AB^2 + ($ �$BC)^2) + (($ �$AB)^2 + BC^2)$

$= AB^2 +$ �$BC^2 + 1/4\ AB^2 + BC^2$

$= 5/4\ (AB^2 + BC^2)$

∴ $4\ (AN^2 + CM^2) = 5AC^2$

Hence proved.

28. Question

If E is a point on side CA of an equilateral triangle ABC such that $BE \perp CA$, then $AB^2 + BC + CA^2 =$

A. 2 BE^2

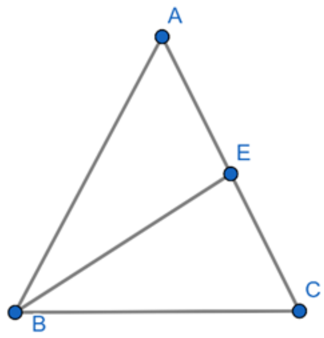
B. 3 BE^2

C. 4 BE^2

D. 6 BE^2

Answer

Given in equilateral ΔABC, BE ⊥ AC.



We know that in an equilateral triangle, the perpendicular from the vertex bisects the base.

∴ CE = AE = AC/2

In ΔABE,

⇒ $AB^2 = BE^2 + AE^2$

Since AB = BC = AC,

⇒ $AB^2 = BC^2 = AC^2 = BE^2 + AE^2$

⇒ $AB^2 + BC^2 + AC^2 = 3BE^2 + 3AE^2$

Since BE is an altitude, $BE = \frac{\sqrt{3}}{2}AB$

$$\Rightarrow BE = \frac{\sqrt{3}}{2}AB$$

$$= \frac{\sqrt{3}}{2}AC = \frac{\sqrt{3}}{2}(2AE)$$

BE = √3 AE

⇒ $AB^2 + BC^2 + AC^2 = 3BE^2 + 3\left(\frac{BE}{\sqrt{3}}\right)^2$

$= 3BE^2 + BE^2$

∴ $AB^2 + BC^2 + AC^2 = 4BE^2$

29. Question

In a right triangle ABC right-angled at B, if P and Q are points on the sides AB and AC respectively, then

A. $AQ^2 + CP^2 = 2(AC^2 + PQ^2)$

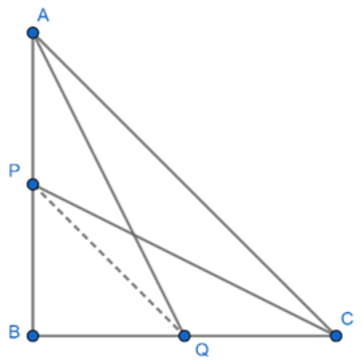
B. $2(AQ^2 + CP^2) = AC^2 + PQ^2$

C. $AQ^2 + CP^2 = AC^2 + PQ^2$

D. AQ + CP= $\frac{1}{2}$ (AC + PQ).

Answer

Given in right triangle ABC right-angled at B, P and Q are points on the sides AB and BC respectively.



Applying Pythagoras Theorem,

In ΔAQB,

⇒ $AQ^2 = AB^2 + BQ^2$ ... (1)

In ΔPBC,

⇒ $CP^2 = PB^2 + BC^2$ ... (2)

Adding (1) and (2),

⇒ $AQ^2 + CP^2 = AB^2 + BQ^2 + PB^2 + BC^2$ ... (3)

In ΔABC,

⇒ $AC^2 = AB^2 + BC^2$ ... (4)

In ΔPBQ,

⇒ $QP^2 = PB^2 + BQ^2$ ... (5)

From (3), (4) and (5),

∴ $AQ^2 + CP^2 = AC^2 + PQ^2$

30. Question

If in ΔABC and ΔDEF , $\frac{AB}{DE} = \frac{BC}{FD}$, then $\Delta ABC \sim \Delta DEF$ when

A. $\angle A = \angle F$

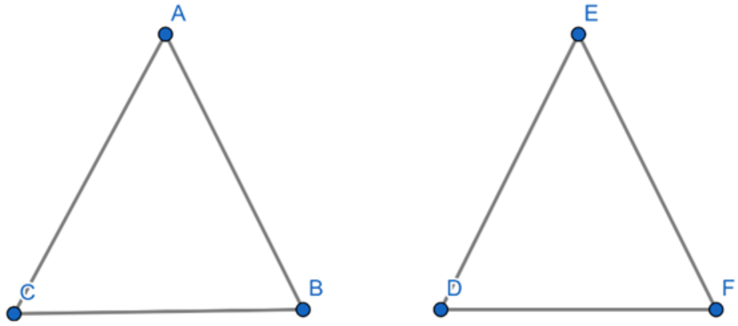
B. $\angle A = \angle D$

C. $\angle B = \angle D$

D. $\angle B = \angle E$

Answer

Given in ΔABC and ΔDEF, $\frac{AB}{DE} = \frac{BC}{FD}$



We know that if in two triangles, one pair of corresponding sides are proportional and included angles are equal, then the two triangles are similar.

Hence, ΔABC is similar to ΔDEF, we should have ∠B = ∠D.

31. Question

If in two triangles ABC and DEF, $\frac{AB}{DE} = \frac{BC}{FE} = \frac{CA}{FD}$, then

A. $\Delta FDE \sim \Delta CAB$

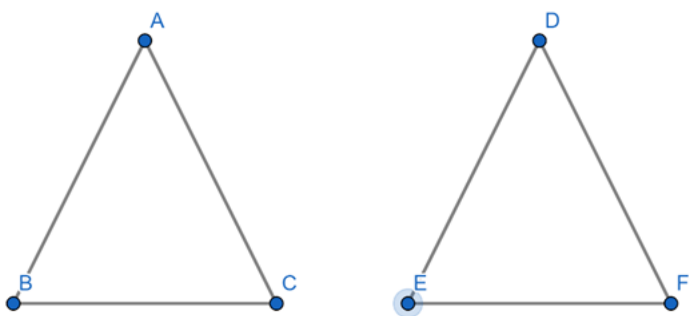
B. $\Delta FDE \sim \Delta ABC$

C. $\Delta CBA \sim \Delta FDE$

D. $\Delta BCA \sim \Delta FDE$

Answer

Given that ΔABC and ΔDEF are two triangles such that $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$



We know that AAA similarity criterion states that in two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

⇒ ∠A = ∠D, ∠B = ∠E, ∠C = ∠F

∴ ΔCAB ~ ΔFDE

Hence proved.

32. Question

$\Delta ABC \sim \Delta DEF$, ar (ΔABC) = 9 cm^2, ar (ΔDEF) = 16 cm^2. If BC = 2.1 cm, then the measure of EF is

A. 2.8 cm

B. 4.2 cm

C. 2.5 cm

D. 4.1 cm

Answer

Given Ar (ΔABC) = 9 cm^2, ar (ΔDEF) = 16 cm^2 and BC = 2.1 cm

We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC^2}{EF^2}$$

$$\Rightarrow \frac{9}{16} = \frac{2.1^2}{EF^2}$$

$$\Rightarrow \frac{3}{4} = \frac{2.1}{EF}$$

∴ EF = 2.8 cm

33. Question

The length of the hypotenuse of an isosceles right triangle whose one side is $4\sqrt{2}$ cm is

A. 12 cm

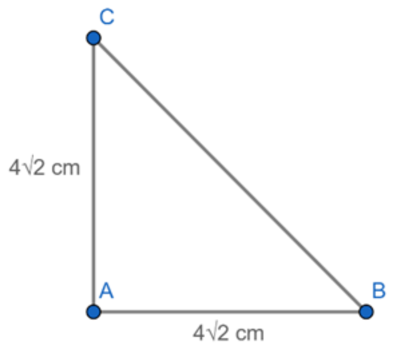
B. 8 cm

C. $8\sqrt{2}$ cm

D. $12\sqrt{2}$ cm

Answer

Given that one side of isosceles right triangle is 4√2 cm.



We know that in isosceles triangle two sides are equal.

In isosceles triangle ABC, let AB and AC be two equal sides of measure $4\sqrt{2}$ cm.

We know that the Pythagoras theorem state that in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Now, in right triangle ABC,

$\Rightarrow BC^2 = AB^2 + AC^2$

$\Rightarrow BC^2 = (4\sqrt{2})^2 + (4\sqrt{2})^2$

$= 32 + 32$

$= 64$

$\therefore BC = 8$ cm

34. Question

A man goes 24 m due west and then 7 m due north. How far is he from the starting point?

A. 31 m

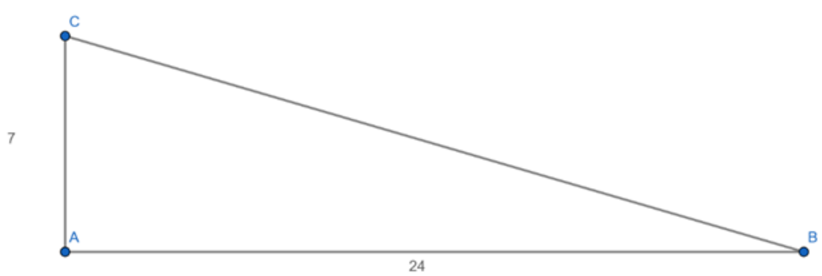
B. 17 m

C. 25 m

D. 26 m

Answer

Given a man goes 24 m due west and then 7 m due north.



We know that the Pythagoras theorem state that in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Now, in right triangle ABC,

⇒ $BC^2 = AB^2 + AC^2$

$= 24^2 + 7^2$

= 576 + 49

= 625

∴ BC = 25 m

35. Question

$\Delta ABC \sim \Delta DEF$. If BC = 3 cm, EF = 4 cm and ar (ΔABC) = 54 cm^2, then ar (ΔDEF) =

A. 108 cm^2

B. 96 cm^2

C. 48 cm^2

D. 100 cm^2

Answer

Given ΔABC ~ ΔDEF, BC = 3 cm, EF = 4 cm and ar (ΔABC) = 54 cm^2

We know that ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

$$\Rightarrow \frac{\text{ar}(\Delta\text{ABC})}{ar(\Delta DEF)} = \frac{BC^2}{EF^2}$$

$$\Rightarrow \frac{54}{ar(\Delta DEF)} = \frac{3^2}{4^2}$$

$$\Rightarrow \frac{54}{ar(\Delta DEF)} = \frac{9}{16}$$

$$\Rightarrow ar(\Delta DEF) = \frac{16(54)}{9}$$

∴ ar (ΔDEF) = 96 cm^2

36. Question

$\Delta ABC \sim \Delta DEF$. such that ar ($\Delta ABC$) = 4 ar ($\Delta PQR$). If BC =12 cm, then QR =

A. 9 cm

B. 10 cm

C. 6 cm

D. 8 cm

Answer

Given ar (ΔABC) ~ ar (PQR) such that ar (ΔABC) = 4 ar (ΔPQR) and BC = 12 cm

We know that ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

$$\Rightarrow \frac{\mathrm{ar}(\Delta \mathrm{ABC})}{ar(\Delta PQR)} = \frac{BC^2}{QR^2}$$

$$\Rightarrow \frac{4ar(\Delta PQR)}{ar(\Delta PQR)} = \frac{12^2}{QR^2}$$

$$\Rightarrow \frac{4}{1} = \frac{12^2}{QR^2}$$

$$\Rightarrow \frac{2}{1} = \frac{12}{QR}$$

∴ QR = 6 cm

37. Question

The areas of two similar triangles are 121 cm^2 and 64 cm^2 respectively. If the median of the first triangle is 12.1 cm, then the corresponding median of the other triangle is

A. 11 cm

B. 8.8 cm

C. 11.1 cm

D. 8.1 cm

Answer

Given areas of two similar triangles 121 cm^2 and 64 cm^2 respectively. The median of the first triangle is 12.1 cm.

We know that ratio of areas of two similar triangles is equal to the ratio of squares of their medians.

$$\Rightarrow \frac{\mathrm{ar}(\Delta 1)}{ar(\Delta 2)} = \frac{median1^2}{median2^2}$$

$$\Rightarrow \frac{121}{64} = \frac{12.1^2}{median2^2}$$

$$\Rightarrow \frac{11}{8} = \frac{12.1}{median2}$$

∴ Median2 = 8.8 cm

38. Question

If $\Delta ABC \sim \Delta DEF$ such that DE = 3 cm, EF = 2 cm, DF = 2.5 cm, BC = 4 cm, then perimeter of ΔABC is

A. 18 cm

B. 20 cm

C. 12 cm

D. 15 cm

Answer

Given that ΔABC and ΔDEF are similar triangles such that AB = 3 cm, DE = 3 cm, DF = 2.5 cm and EF = 2 cm.

We know that two triangles are similar if their corresponding sides are proportional.

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$

First consider,

$$\frac{AB}{BC} = \frac{DE}{EF}$$

$$\Rightarrow \frac{AB}{4} = \frac{3}{2}$$

⇒ AB = 6 cm … (1)

Now,

$$\frac{CA}{BC} = \frac{DF}{EF}$$

$$\Rightarrow \frac{CA}{4} = \frac{2.5}{2}$$

⇒ CA = 5 cm … (2)

Then, perimeter of ΔABC = AB + BC + CA = 6 + 4 + 5

∴ Perimeter of ΔABC = 15 cm

39. Question

In an equilateral triangle ABC if $AD \perp BC$, then AD^2 =

A. CD^2

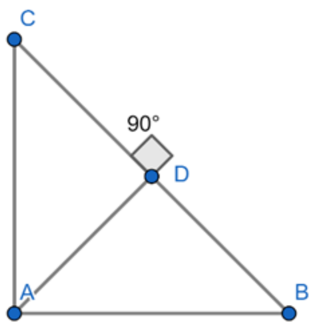
B. $2CD^2$

C. $3CD^2$

D. $4CD^2$

Answer

Given in equilateral triangle ABC, AD ⊥ BC.



We know that the Pythagoras theorem state that in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Now, in right triangle ADC,

$\Rightarrow AC^2 = AD^2 + DC^2$

$\Rightarrow BC^2 = AD^2 + DC^2$ [$\because$ AC = BC]

$\Rightarrow (2DC)^2 = AD^2 + DC^2$ [$\because$ BC = 2DC]

$\Rightarrow 4DC^2 = AD^2 + DC^2$

$\Rightarrow 3DC^2 = AD^2$

$\therefore 3CD^2 = AD^2$

40. Question

In an equilateral triangle ABC if $AD \perp BC$, then

A. $5AB^2 = 4AD^2$

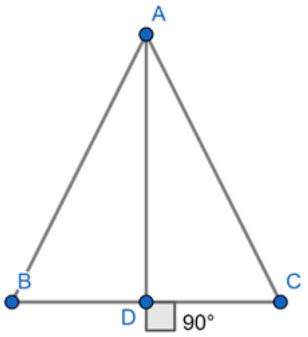
B. $3AB^2 = 4AD^2$

C. $4AB^2 = 3AD^2$

D. $2AB^2 = 3AD^2$

Answer

Given in equilateral triangle ABC if AD ⊥ BC.



We know that the Pythagoras theorem state that in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Now, in right triangle ABD,

⇒ $AB^2 = AD^2 + BD^2$

⇒ $AB^2 = AD^2 +$ (� $BC)^2$ [∵ BD = �BC]

⇒ $AB^2 = AD^2 +$ (� $AB)^2$ [∵ AB = BC]

⇒ $AB^2 = AD^2 +$ (�$AB)^2$

∴ $3AB^2 = 4AD^2$

41. Question

If $\Delta ABC \sim \Delta DEF$ such that AB = 9.1 cm and DE = 6.5 cm. If the perimeter of ΔDEF is 25 cm, then the perimeter of ΔABC is

A. 36 cm

B. 30 cm

C. 34 cm

D. 35 cm

Answer

Given ΔABC ~ ΔDEF such that AB = 9.1 cm and DE = 6.5 cm.

Given that ΔABC and ΔDEF are similar triangles such that AB = 3 cm, BC = 2 cm, CA = 2.5 cm and EF = 4 cm.

We know that ratio of corresponding sides of similar triangles is equal to the ratio of the perimeters.

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = \frac{P1}{P2}$$

Consider,

$$\frac{AB}{DE} = \frac{P(\Delta ABC)}{P(\Delta DEF)}$$

$\Rightarrow \frac{9.1}{6.5} = \frac{P(\Delta ABC)}{25}$

∴ P(ΔABC) = 35 cm

42. Question

In an isosceles triangle ABC if AC = BC and $AB^2 = 2AC^2$, then $\angle C =$

A. 30°

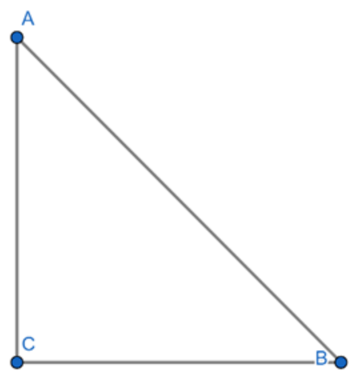
B. 45°

C. 90°

D. 60°

Answer

Given in isosceles ΔABC, AC = BC and $AB^2 = 2AC^2$



In isosceles ΔABC,

AC = BC, so ∠B = ∠A [Equal sides have equal angles opposite to them]

$\Rightarrow AB^2 = 2AC^2$

$\Rightarrow AB^2 = AC^2 + AC^2$

$\Rightarrow AB^2 = AC^2 + BC^2$

∴ ΔABC is right angle triangle with ∠C = 90°

43. Question

ΔABC is an isosceles triangle in which $\angle C = 90°$. If AC = 6 cm, then AB=

A. $6\sqrt{2}$ cm

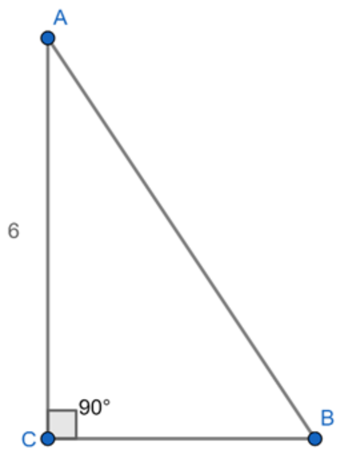
B. 6 cm

C. $2\sqrt{6}$ cm

D. $4\sqrt{2}$ cm

Answer

Given in an isosceles triangle ABC, ∠C = 90° and AC = 6 cm.



⇒ BC = AC = 6 cm

We know that the Pythagoras theorem state that in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Now, in right triangle ABC,

⇒ $AB^2 = AC^2 + BC^2$

$= 6^2 + 6^2$

= 36 + 36

= 72

∴ AB = 6√2 cm

44. Question

If in two triangles ABC and DEF, $\angle A = \angle E$, $\angle B = \angle F$, then which of the following not true?

A. $\frac{BC}{DF} = \frac{AC}{DE}$

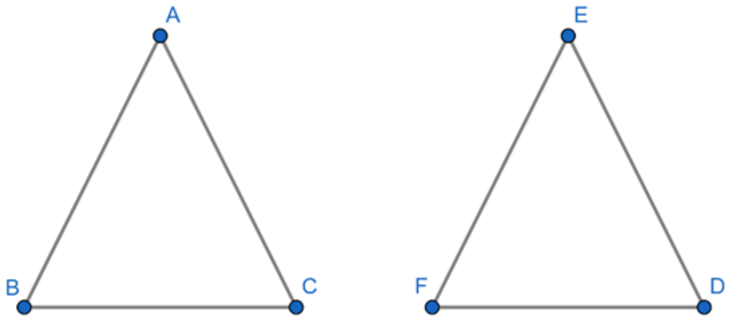
B. $\frac{AB}{DE} = \frac{BC}{DF}$

C. $\frac{AB}{EF} = \frac{AC}{DE}$

D. $\frac{BC}{DF} = \frac{AB}{EF}$

Answer

Given that ΔABC and ΔDEF are two triangles such that ∠A = ∠E and ∠B = ∠F.



We know that AAA similarity criterion states that in two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

$$\Rightarrow \frac{AB}{EF} = \frac{BC}{FD} = \frac{CA}{DE}$$

∴ ΔABC ~ ΔDEF

Hence proved.

45. Question

In an isosceles triangle ABC, if AB = AC = 25 cm and BC = 14 cm, then the measure of altitude from A on BC is

A. 20 cm

B. 22 cm

C. 18 cm

D. 24 cm

Answer

Given in an isosceles ΔABC, AB = AC = 25 cm and BC = 14 cm

Here altitude from A to BC is AD.

We know in isosceles triangle altitude on non-equal sides is also median.

⇒ BD = CD = BC/2 = 7 cm

Applying Pythagoras Theorem,

⇒ $AB^2 = BD^2 + AD^2$

⇒ $25^2 = 7^2 + AD^2$

⇒ $AD^2 = 625 - 49 = 576$

⇒ AD = 24

∴ Measure of altitude from A to BC is 24 cm

46. Question

In Fig. 4.242 the measures of $\angle D$ and $\angle F$ are respectively

A. 50°, 40°

B. 20°, 30°

C. 40°, 50°

D. 30°, 20°

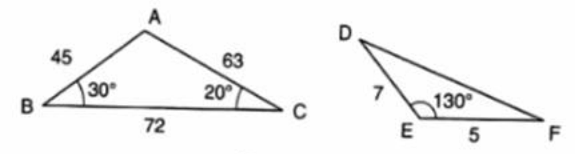


Fig. 4.242

Answer

In ΔABC and ΔDEF,

$$\Rightarrow \frac{AB}{AC} = \frac{EF}{ED}$$

⇒ ∠A = ∠E = 130°

We know that SAS similarity criterion states that if one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

∴ ΔABC ~ ΔEFD

Hence, ∠F = ∠B = 30°

And ∠D = ∠C = 20°

47. Question

In Fig. 4.243, the value of x for which $DE \parallel AB$ is

A. 4

B. 1

C. 3

D. 2

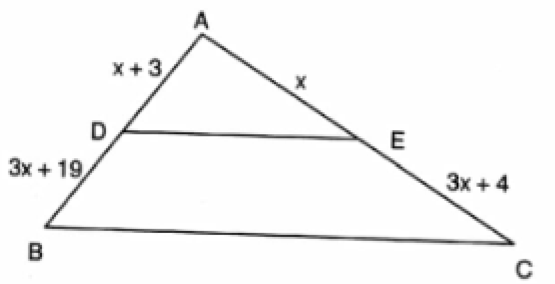


Fig. 4.243

Answer

Given in ΔABC, DE || AB.

We know that basic proportionality theorem states that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

Then $\frac{AD}{DB} = \frac{AE}{EC}$

$$\Rightarrow \frac{x+3}{3x+19} = \frac{x}{3x+4}$$

⇒ (x + 3) (3x + 4) = x (3x + 19)

⇒ $3x^2 + 4x + 9x + 12 = 3x^2 + 19x$

⇒ 19x − 13x = 12

⇒ 6x = 12

∴ x = 2 cm

48. Question

In Fig. 4.244, if $\angle ADE = \angle ABC$, then CE =

A. 2

B. 5

C. 9/2

D. 3

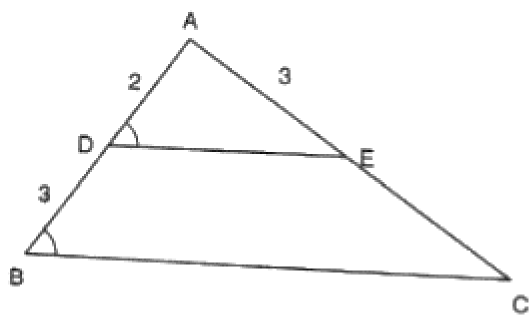


Fig. 4.244

Answer

Given ∠ADE = ∠ABC

We know that basic proportionality theorem states that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

Then $\frac{AD}{DB} = \frac{AE}{EC}$

$$\Rightarrow \frac{2}{3} = \frac{3}{EC}$$

$$\Rightarrow EC = \frac{3(3)}{2}$$

∴ EC = 9/2 cm

49. Question

In Fig. 4.245, $RS \| DB \| PQ$. If CP = PD =11 cm and DR = RA = 3 cm. Then the values of x and y are respectively

A. 12, 10

B. 14, 6

C. 10, 7

D. 16, 8

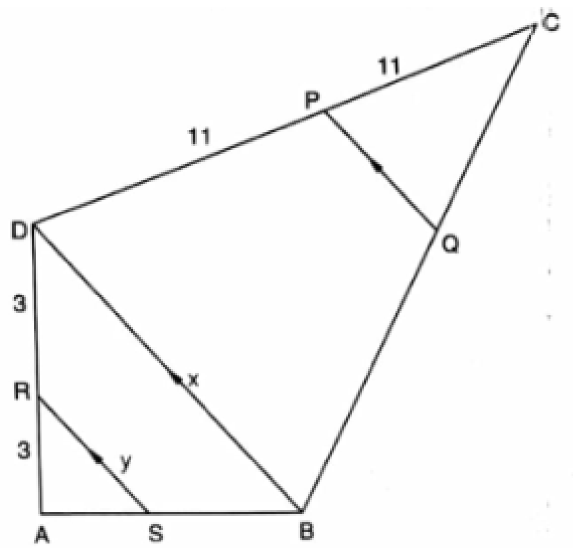


Fig. 4.245

Answer

Given in figure RS || DB || PQ, CP = PD = 11 cm and DR = RA = 3 cm.

In ΔASR and ΔABD,

∠ASR = ∠ABD [corresponding angles]

∠ARS = ∠ADB [corresponding angles]

∠A = ∠A [common]

We know that AAA similarity criterion states that in two triangles, if corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

∴ ΔASR ~ ΔABD

We know that two triangles are similar if their corresponding sides are proportional.

$$\Rightarrow \frac{AR}{AD} = \frac{AS}{AB} = \frac{RS}{DB}$$

$$\Rightarrow \frac{3}{6} = \frac{RS}{DB}$$

$$\Rightarrow \frac{1}{2} = \frac{x}{y}$$

∴ x= 2y

∴ x = 16 cm and y = 8 cm

50. Question

In Fig. 4.246, if $PB \| CF$ and $DP \| EF$, then $\frac{AD}{DE}$ =

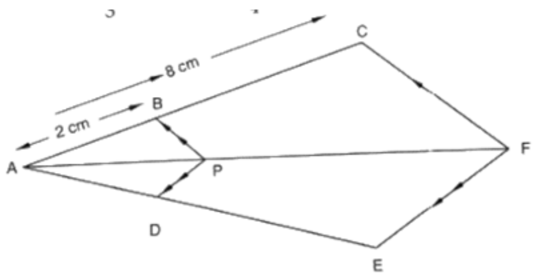


Fig. 4.246

Answer

Given PB || CF, DP || EF, AB = 2 cm and AC = 8 cm

We know that basic proportionality theorem states that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

In ΔACF, PB || CF,

Then $\frac{AB}{BC} = \frac{AP}{PF}$

$$\Rightarrow \frac{AP}{PF} = \frac{2}{8-2} = \frac{2}{6} = \frac{1}{3}$$

And DP || EF

$$\Rightarrow \frac{AD}{DE} = \frac{AP}{PF}$$

$$\therefore \frac{AD}{DE} = \frac{1}{3}$$

51. Question

A chord of a circle of radius 10 cm subtends a right angle at the centre. The length of the chord (in cm) is

A. $5\sqrt{2}$

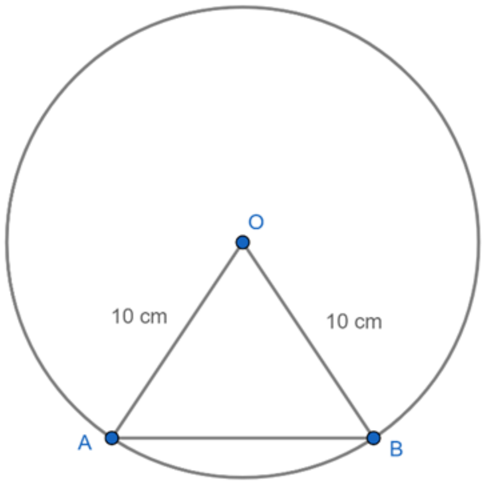
B. $10\sqrt{2}$

C. $\frac{5}{\sqrt{2}}$

D. $10\sqrt{3}$ [CBSE 2014]

Answer

Given A chord of a circle of radius 10 cm subtends a right angle at the centre.



We know that the Pythagoras theorem state that in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Now, in right triangle OAB,

$\Rightarrow AB^2 = OA^2 + OB^2$

$= 10^2 + 10^2$

$= 100 + 100$

$= 200$

$\therefore AB = 10\sqrt{2}$ cm